ANALYSIS OF LATERALLY LOADED PILES UNDER LIQUEFIED SOIL CONDITIONS

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by

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CERTIFICATE

It is certified that the work contained in the thesis entitled, "ANALYSIS OF LATERALLY LOADED PILES UNDER LIQUEFIED SOIL CONDITIONS" by Sudhir Kumar Mishra, has been carried out under my supervision and that this work has not been submitted elsewhere for a degree.

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ABSTRACT

In this thesis an attempt has been made to predict the utlimate lateral load capacity and the flexural behaviour of a pile when a certain depth of soil along the length of the pile has liquefied. The analysis has been carried out assumming that either there is complete or partial loss of soil support due to soil liquefaction over the estimated zone and computer programs have been developed for the same. The obtained results have been presented in the form of non-dimensional design charts for both short and long piles for various conditions of pile head fixity and ground conditions.

The study brings out both qualitatively and quantitatively the significant influence of the liquefied soil zone on the lateral load capacity and bending behaviour of piles. The study also reveals that the presence of non-liquefied soil cover increases the load capacity and decreases the pile top deflection in comparison to the corresponding values when the liquefaction occurs starting from the ground surface. This highlights the importance of soil densification in the upper zone of the soil.

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Sudhir Kumar Mishra

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SYMBOLS

Am	Bending Moment Coefficient
Ay	Deflection Coefficient
d	Diameter of Pile, m
e	Eccentricity of load from the ground surface, m
E	Modulus of Elasticity of Pile, kN/m^2
FD	Drag Force, kN
H	Lateral load, kN
H_u	Ultimate Lateral Load, kN
I	Second Moment of Area of the Pile Section, m^4
k_h	Modulus of Horizontal subgrade Reaction, kN/m^3
K_p	Coefficient of Passive Earth Pressure of Soil
l	Embedded Length of the Pile, m
p	Soil Pressure, kN/m^2
T	Relative Stiffness Factor, m
w	Horizontal Pile Deflection, m
Z	Depth Coefficient
ρ	Density of Liquefied Soil, kg/m^3
γ	Unit Weight of Soil, kN/m^3

1. INTRODUCTION

Liquefaction of saturated sands during earthquake is the cause of much damage to Civil Engineering Structures. Several instances of such damages have been observed in Niigata earthquake and Alaska earthquake that occured in 1964. The depth upto which a soil is likely to liquefy is governed by the ground water conditions, soil type, the earthquake magnitude and its epicentral distance from the concerned site. The liquefied depth may be estimated from different semi-empirical predictive models based on simple theories and experiments conducted in the laboratory and insitu to predict the liquefaction potential of a site. Some of these models are due to Seed et al. (1971), Lee et al (1972), Shibata et al. (1972), Ishihara and Yashuda (1972), Yoshmi and Kuwabara (1973), Yoshmi and Oka (1975), etc.

Once the probable depth of liquefaction is found out it can be used as an input data to estimate the effect of this liquefied zone on the existing foundations. Depending on the risks of soil liquefaction at a site different remedial measures like vibrofloatation, dynamic compaction etc for improving the ground condition may be adopted. Compaction of submerged sand to a relative density of 70 to 80 percent is adequate enough to prevent liquefaction. Alternatively the load bearing piles may be designed and adopted considering the effect of soil liquefaction on the pile capacity and the flexural behaviour under lateral loads. However, economy would be the main consideration in making the proper choice.

Very often the engineers are called upon to make a comparative assessment of the various options available for making a final decision. Ground improvement techniques and processes have been studied over the years and design charts are available in literature. Considerable information on these techniques are now available in standard text books (van Impe, 1989; Hausman, 1990) and reference books (Balasubramaniam et al. 1985). Even though there are many theories available to predict the liquefaction potential at a site apart from damage survey the literature pertaining to the prediction of the effect of the liquefied depth on the foundation performance is scanty (Miura and O'Rouke, 1991).

For the cases when there is no soil liquefaction the ultimate lateral load capacity of piles may be estimated by using the methods proposed by Hansen (1961) and Broms (1964 (a) and 1964 (b)), Roy (1970), Meyerhof et al. (1981). Some of these methods are now available in standard text books on foundation engineering (Tomlinson, 1980; Poulos and Davis, 1980). Viggiani (1991) investigated the effect of sliding soil mass caused by land slides on the lateral load capacity considering various failure modes of the pile. The method has the potential to be modified and used in predicting the lateral load capacity of pile in liquefied soil deposit.

The flexural analysis of a laterally loaded pile is generally carried out by using any one of the following approaches:

(i) Matlock and Reese approach (1960) using the concept of modulus of subgrade reaction.

(ii) Poulos and Davis approach (1980) using the Mindlin

solution.

(iii) Finite element method alongwith appropriate constitutive modelling e.g. Desai and Abel (1972), Chen and Baladi (1985).

Details of most of these methods have been presented by Poulus and Davis (1980), Zienkiewicz (1975), Desai and Christian (1977) and as such these are not reviewed here.

It is evident that even though the detrimental effect of the loss of support due to soil liquefaction over a certain portion of an embedded pile is well recognised, the attention of geotechnical research community has not been focussed to quantify them. As such in this thesis an effort has been made to study the effect of liquefied soil zone on the ultimate lateral load capacity and flexural behaviour of a laterally loaded pile.

2. STATEMENT OF THE PROBLEM

As shown in figure 2.1, a pile of length l and diameter d is embedded in a homogeneous saturated sand deposit. Considering the worst condition the water table has been assummed to be at the ground level. The depth of the soil cover which has not liquefied is l_1. Over the depth l_2, soil is in a liquefied state and the pile has lost the support of the soil either completely or partially. In the lower portion of the pile over a length l_3 the soil is again non-liquefied. The sandy soil deposit is having an average submerged unit weight of γ and the effective angle of shearing resistance ϕ'. The horizontal load at the top of the pile is acting at a height e from the ground surface. The yield moment of the pile section is M_y.

In this problem the possible movement of liquefied soil in the liquefied zone has also been considered. It has been assumed that the liquefied soil flows with a maximum velocity of V_o m/s at the top end of the liquefied zone as shown in the figure and lineraly varies to zero at its lower end. This flowing mass of soil will cause a drag force on the pile. However, the velocity of the soil movement is likely to be small.

Thus as shown in the figure, for given inputs of l_1, l_2, l_3, e and soil and pile properties, the objective is to determine the ultimate lateral load capacity of the pile,H_u,and the flexural behaviour of the pile when it has completely lost support over the specified length due to soil liquefaction.

Liquefied Soil Condition

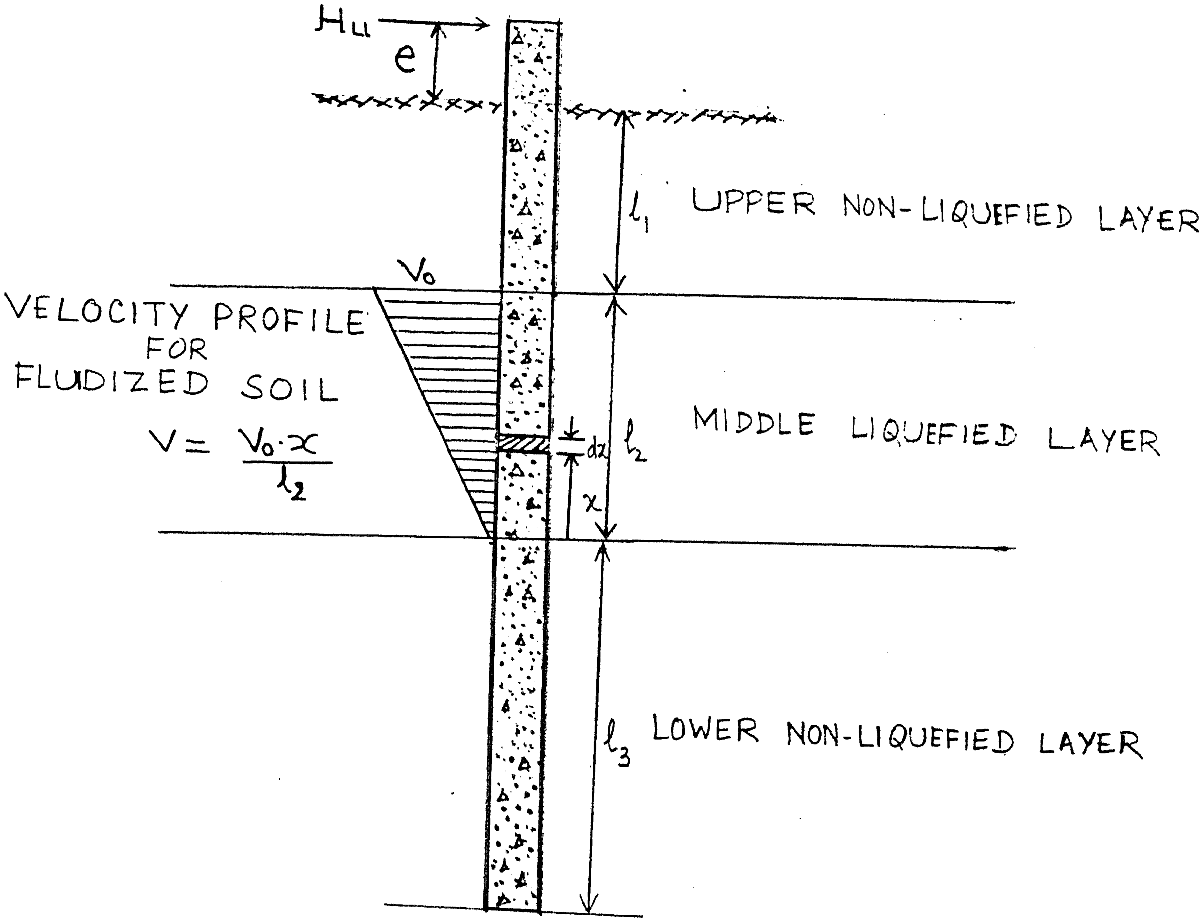


FIG.2·1

3. ANALYSIS

3.1 Ultimate Lateral Load Capacity

In estimating the lateral load capacity the intensity of earth pressure acting at a point in a cohesionless medium due to the lateral displacement of the pile has been calculated by extending the Brom's theory (1964) and assuming that earth pressure intensities at any depth in the non-liquefied zone can still be estimated by using the Brom's approach.

As shown in the figure 2.1, the input data for the ultimate lateral load capacity of the pile are following:

l_1 = depth of upper non-liquefied layer

l_2 = depth of middle liquefied layer

l_3 = depth of lower non-liquefied layer

K_p = coefficient of passive earth pressure of soil

γ = effective unit weight of the soil deposit

ρ = density of the fluidised soil

d = diameter of the pile

M_y = yield moment of the pile.

The various forces acting on a pile and their direction are shown in the figure 3.1. These are the forces when the failure is due to the yielding of the soil.

a = soil resistance due to upper non-liquefied layer

$= 1/2 * 3\gamma\, d\, K_p\, l_1 x l_1$

$= 1.5\,\gamma\, d\, K_p\, l_1^2$ acting at $2l_1/3$ from the ground surface

b = soil resistance (due to rectangular portion) for the lower non-liquefied layer.

$= 3\gamma d K_p (l_1 + l_2) . l_3$ at $l_3/3$ from the upper end of l_3 layer

c = soil resistance due to triangular part

$= 1/2 * 3\gamma d K_p l_3 . l_3$

$= 1.5\gamma d K_p l^2{}_3$ at $2.l_3/3$ from the upper end.

With reference to Fig. 2.1, at any depth in the liquefied zone, the drag force FD can be computed by using the drag force equation for fluid mass as :

$$FD = 1/2 \rho V^2 C_D . A$$

where V = velocity of flow of the liquefied soil

C_D = drag coefficient

A = Projected area of the pile

ρ = density of fluidised soil

Here $V = V_o x/ l_2$

For an elementary length dx at distance x from the lower end

$$d(FD) = 1/2 \rho V^2 C_D.A$$
$$= 1/2 \rho (V_o^2 . x^2/ l_2^2) . C_D . d . dx$$

$$FD = 1/2 \rho \frac{V_o^2 . C_D.d}{l^2{}_2} \int_o^{l_2} x^2.dx$$

$$= \frac{\rho V_o^2.C_D.d}{2l_2^2} . \frac{l_2^3}{3}$$

$$= \frac{\rho V_o^2 d l_2.C_D}{6}$$

If the location of the centroid of the force be at x_2 from the lower end of the liquefied zone then

$$FD . x_2 = \int d(FD).x = \int_o^{l_2} 1/2 \rho V_o \frac{x^2}{l^2} . C_D.d.dx.x$$

$$\frac{\rho V_o^2 \cdot d \cdot C_D \cdot l_2 \cdot x_2}{6} = 1/2 \frac{\rho V_o^2 \quad C_D \cdot d}{l_2^2} \cdot \frac{l_2^4}{4}$$

$$x_2 = 0.75\ l_2$$

Thus the drag force $FD = \dfrac{\rho V_o^2\ d.l_2.C_D}{6}$ acts at 0.75 l_2 from the bottom end of the liquefied zone.

Now, using the above forces the estimation for the ultimate lateral capacity of pile can be made. The lateral load capacity for free head and fixed head piles have been considered separately.

3.1.1 <u>Free Head Pile</u> : To start with it is assumed that the pile behaves as a short pile and it fails due to the failure of soil only. Here the maximum bending moment does not reach the yield moment of the pile section.

The ultimate lateral load capacity of the pile for this case can be estimated by taking the moment of all the forces about the lower end of the pile (Fig. 3.1):

$$H_u\ (e+l) + FD(0.75\ l_2 + l_3) = a(\frac{l_1}{3} + l_2 + l_3) + b.\frac{l_3}{2} + c.\frac{l_3}{3}$$

$$Hu = \{a(l-0.67\ l_1) + b.l_3/2 + c.l_3/3 - FD(0.75\ l_2+l_3)\}\ /\ (e+l) \qquad -(3.1.1)$$

To check the validity of the assumption that the failure is not due to the development of plastic hinge in the pile, a check as follows is introduced in the analysis.

If the maximum moment occurs in the l_1 part and f is the depth of zero shear force below the ground surface then

$$H_u = 1.5 \gamma d Kpf^2$$

or $$f = 0.82 \left(\frac{H_u}{\gamma dKp}\right)^{1/2} \tag{3.1.2}$$

Thus if $f < l_1$ then the maximum moment occurs in the upper part, given by $M_{max} = H_u (e + 2/3 f)$.

Now, if $M_{max} < M_y$ then the pile is behaving as a short pile. On the other hand if $M_{max} \geqslant M_y$ then the pile behaves as a long pile.

For the short pile case the ultimate lateral load capacity of pile is given by equation (3.1.1).

For the long pile case the ultimate lateral resistance is given by the equations

$$M_y = H_u (e + 2/3 f) \tag{3.1.3}$$

$$H_u = 1.5 \gamma dKpf^2 \tag{3.1.4}$$

If f calculated from equation (3.1.2) is such that $f > l_1$ then the maximum moment will be in part l_3.

Let the maximum moment occur at a distance x from the lower end of the liquefied zone as shown in the figure 3.2.

The condition for maximum moment is that net shear force should be zero. Referring to Figure 3.2,

$$H_u + FD = 1.5 \gamma dKpl_1^2 + 3 \gamma dKp (l_1 + l_2) . x + 1.5 \gamma dK_p x^2$$

(here H_u given from equation 3.1.1)

The maximum bending moment occuring at the zero shear depth is computed by taking moment of forces about that point.

$$M_{max} = H_u (e+l_1+l_2+x) + FD (0.75 l_2 + x) - a (l_1/3+l_2+x)$$
$$-3 \gamma dK_p(l_1 + l_2).x.x/2 - 1.5 \gamma dK_p x^2.x/3$$

$$M_{max} = H_u(e+l_1+l_2+x) + FD\ (0.75l_2+x) - a(l_1/3 + l_2 + x) \\ -1.5\,\gamma\, dK_p\ (l_1+l_2)x^2 - 0.5\,\gamma\, dK_p x^3$$

Now again, if $M_{max} < M_y$ then the pile behaves as a short pile an its ultimate lateral capacity is given by equation (3.1.1).

But if $M_{max} \geqslant M_y$ then the pile behaves as a long pile. For this case the ultimate lateral capacity will be given by the solution of two equations viz.

$$M_y = H_u\ (e + l_1+l_2+x) + FD(0.75l_2 + x) - a(l_1/3 + l_2 + x) \\ - 1.5\,\gamma\, dKp\ (l_1+l_2)x^2 - 0.5\,\gamma\, dKpx^3 \qquad (3.1.5)$$

and $Hu + FD = 1.5\,\gamma\, dKpl_1{}^2 + 3\,\gamma\, dKp\ (l_1 + l_2)x + 1.5\,\gamma\, dKpx^2$

$$- \ (3.1.6)$$

The above two equations can be solved by an iterative method to give the ultimate lateral load capacity of the pile. This will also give the depth at which the yielding occurs.

3.1.2 <u>Fixed Head Pile</u> : Assuming that the fixed head pile is behaving as a short pile, from fig.3.3 for the horizontal equilibrium of the pile

$$Hu = a + b + c - FD$$

where a,b,c have the same meaning as described earlier.

$$Hu = 1.5\,\gamma\, dKpl_1{}^2 + 3\,\gamma\, dKp(l_1+l_2).l_3 + 1.5\gamma\ dKpl_3{}^2 - FD \quad .(3.1.7)$$

In fixed pile case, the maximum moment occurs at the fixed end. Thus taking moment of all the forces about the fixed end

$$M_{max} = a(2/3\ l_1+e) + b(e+l-l_3/2) + c\ (e+l-l_3/3) \\ - FD(0.25l_2 + l_1+e) \qquad (3.1.8)$$

Now, if $M_{max} < M_y$ then the pile is behaving as a short pile and the ultimate lateral load capacity is given by equation (3.1.7).

If $M_{max} \geq M_y$ then the pile is behaving as an intermediate pile and the yield occurs first at the fixed end.

Now from force equlibrium (Fig.3.3).

$$F + Hu + FD = a + b + c$$

where F is the reaction at the lower end of the pile and a,b,c and FD have the same meaning as described earlier.

$$F = a + b + c - FD - Hu \qquad (3.1.9)$$

Thus taking moment of all the forces about the fixed end of the pile.

$$M_y = a(2/3\ l_1+e) + b(e+l-l_3/2) + c(e+l-l_3/3) - FD(0.25l_2 + l_1 + e) - F(l+e)$$

From equation (3.1.8) the above equation can be written in a convenient form as

$$M_y = M_{max} - F(l+e) \qquad (3.1.10)$$

Now from equations (3.1.8), (3.1.9) and (3.1.10) the ultimate lateral capaity of the pile for the intermediate case can be computed.

If the fixed head pile behaves as a long pile then another plastic hinge will form at some other point away from the fixed end.

Now, if the second yield occurs in the l_1 part then taking moment about that point

$$Hu\ (e + 2/3\ f) = 2M_y \qquad (3.1.11)$$

where $f = 0.82 \left(\frac{Hu}{\gamma dKp}\right)^{1/2}$ (from zero shear force) (3.1.12)

Thus the solution of these two equations (3.1.11) and (3.1.12) will give the value of ultimate lateral resistance for the long pile case. If the above calculated value of $f > l_1$ then the second yield may occur in the l_3 part.Again taking

moment about that point which is at a distance x below the liquefied zone (Fig. 3.4)

$$Hu\ (e+l_1+l_2+x) - a(l_1/3+l_2+x) - 3\gamma dKp(l_1+l_2) \cdot x^2/2$$
$$- 1.5\gamma\ d\ Kp\ x^3/3 - FD(0.75\ l_2 + x) = 2M_y \qquad (3.1.13)$$

The condition for maximum moment at that point gives

$$Hu + FD = a + 3\gamma d\ Kp(l_1+l_2)x + 1.5\gamma d\ Kp\ x^2 \qquad (3.1.14)$$

The solution of these two equations (3.1.13) and (3.1.14) gives the ultimate lateral capacity for the long pile case.

Based on the expressions as deraved above a computer program has been developed to find the ultimate lateral load capacity of the pile when soil over a certain depth along the embedded lengh of the pile has liquefied. Using the program one can evaluate the lateral load capacity for both free and fixed head pile case and further find the failure mode of the pile i.e. whether it is failing as a short or as a long pile.

For the purpose of providing non-dimensional design charts the drag force due to the possible movement of the liquefied soil has not been taken into account as the contribution due to drag force on the pile capacity is generally negligibly small. This can be quantitatively seen in the example as follows:

For a free head short pile the expression for ultimate capacity is given by

$$Hu = \frac{a(1-0.67l_1) + b\ l_3/2 + 1/3\ c.l_3 - FD(0.75\ l_2+l_3)}{(e + 1)}$$

where $a = 1.5\gamma d\ Kp\ l_1{}^2$

$b = 3\gamma d\ Kp\ (l_1+l_2)l_3$

$c = 1.5\gamma d\ Kp\ l_3{}^2$

$$FD = \frac{\rho V_o^2 \; dl_2 \; C_D}{6}$$

Taking $l_1 = 2$ m

$l_2 = 2$ m

$l_3 = 1$ m

$l = 5$ m

$\gamma = 6$ kN/m^3

$d = 0.5$ m

$e = 0.5$ m

$K_p = 3.0$

$\rho = 1800$ kg/m^3

$C_D = 0.4$

$a = 1.5 \times 6 \times 0.5 \times 3 \times 4 = 54$ kN

$b = 3 \times 6 \times 0.5 \times 3 \times 4 \times 1 = 108$ kN

$c = 1.5 \times 6 \times 0.5 \times 3 \times 1 = 13.5$ kN

$$F_D = 1800 \times V_o^2 \times 0.5 \times 2 \times 0.4 \times \frac{9.8}{1000} \text{ kN}$$

$$= 7.06 \; V_o^2$$

$$Hu = \frac{54(5-0.67x2) + 108x1/2 + 2/3x13.5x1 - 7.06V^2(0.75x2+1)}{5.5}$$

$$= \frac{260.6 - 17.6 \; V^2}{5.5} = 47.4 - 3.2 \; V_o^2$$

Taking $V_o = 0.5$ m/s as soil movement is not likely to be very high.

$$Hu = 47.4 - 3.2 \times .5^2 = 46.6 \text{ kN}$$

Thus we see that Hu value decreases from 47.4 to 46.6 only. Thus the drag force can be neglected without much error.

With a view to present non-dimensional design charts the equations as derived are non-dimensionalised as follows :-

3.1.3 Non-Dimensional Form of the Equations

Case I : Free Head Short Pile

As previously derived the lateral load capacity for a free head short pile the expression is :

$$H_u(e + l) = 1.5\,\gamma d\, Kp\, l_1^2\,(l - 2/3\, l_1) + 1.5\,\gamma d\, Kp\,(l_1 + l_2) l_3^2 + 0.5\,\gamma d\, Kp\, l_3^3$$

But $l_3 = l - l_1 - l_2$

dividing both sides by $\gamma dKp\, l^2$

$$\frac{H_u}{\gamma dK_p l^2} = \frac{1.5\,(l_1/l)^2\,(1-0.67\, l_1/l) + 1.5\,(l_1/l + l_2/l).(1 - l_1/l - l_2/l)^2 + 0.5\,(1 - l_1/l - l_2/l)^3}{(e/l + 1)}$$

Case II - Fixed Head Short Pile

$$H_u = 1.5\,\gamma dK_p\, l_1^2 + 3\,\gamma dKp\,(l_1+l_2) l_3 + 1.5\,\gamma d\, Kp l_3^2$$

Since $l_3 = l - l_1 - l_2$.

$$\frac{H_u}{\gamma dKpl^2} = 1.5\,(l_1/l)^2 + 3(l_1/l + l_2/l)\,(1 - l_1/l - l_2/l) + 1.5\,(1 - l_1/l - l_2/l)^2$$

Case III - Free Head Long Pile

For the case of $l_1 = 0$, from the previous equations

$$H_u = 3\,\gamma d\, K_p\, l_2 x + 1.5\,\gamma dK_p\, x^2$$

and $$My = H_u\,(e + l_2 + x) - 1.5\,\gamma dK_p\, l_2 x^2 - 0.5\,\gamma dK_p x^3$$

$$\frac{H_u}{\gamma dK_p l^2} = 3\,(l_2/l)\,(x/l) + 1.5\,(x/l)^2$$

$$\frac{M_y}{\gamma d K_p l^3} = \frac{H_u}{\gamma d K_p l^2} (e/l + l_2/l + x/l) - 1.5 (l_2/l) (x/l)^2 - 0.5 (x/l)^3$$

Case IV - Fixed Head Long Pile

For $l_1=0$ condition the equations are

$$2 M_y = H_u(e+l_2+x) - 1.5 \gamma dKpl_2x^2 - 0.5 \gamma dKpx^3$$

and $$H_u = 3 \gamma dK_p l_2x + 1.5 \gamma dK_p x^2$$

Thus

$$\frac{2M_y}{\gamma d K_p l^3} = \frac{H_u}{\gamma d K_p l^2} (e/l + l_2/l + x/l) - 1.5 l_2/l(x/l)^2 - 0.5 (x/l)^3$$

and $$\frac{H_u}{\gamma d K_p l^2} = 3 l_2/l (x/l) + 1.5 (x/l)^2.$$

Thus for the given parameters of l_2/l and e/l the above equations can be solved to give the non dimensional lateral load capacity.

3.2 Flexural Behaviour of the Pile

Once the lateral load capacity of a pile is estimated for a given ground conditions the safe design load can be calculated by using a proper factor of safety and the flexural response of the pile can be analysed (Reese and Desai,1977;Poulos and Davis,1980)for its structural design. The adopted analysis procedure is briefly described as follows :

For deflection calcualtion the subgrade reaction analysis based on Winkler Soil model has been adopted(Poulos and Davis,1980)

In the Winkler soil model the pressure p and deflection

w at a point are assumed to be related through a modulus of subgrade reaction, which for a horizontal loading is denoted as k_h.

Thus $p = k_h w$. where k_h has the unit of force/length^3.The pile is assummed to act as a thin strip whose behaviour is governed by the beam equation $EI \dfrac{d^4w}{dx^4} = -p.d.$

where E = modulus of elasticity of pile.

I = moment of inertia of pile section.

x = depth in soil.

d = diameter or width of pile.

Thus the governing equation of a laterally loaded pile is

$$EI \frac{d^4w}{dx^4} + k_h d.w = 0.$$

The solution to above equation can be most conventiently obtained by finite difference method (Bowles, 1988). The analytical solution are only available in convenient forms for the cases of constant and linear variation of k_h along the pile. But in the case of cohesionless soil the value of k_h is not constant and generally varies with depth. For homogeneous cohesionless soil deposits k_h generally increases linearly with depth. To take care of the arbitrary variation of k_h in the soil deposits it is convenient to correlate it with the SPT, N values. In this respect the variation of coefficient of subgrade reaction as proposed by the Japanese Highway Bridge Code (Yoshida and Hamada,1991) for such deposits has been adopted in the analysis as

$k_h = 0.2 \times 28\ N\ d^{(-0.35)}$ (kgf/cm^3) where d denotes the diameter of the pile in cm and N denotes the SPT value.

set one tenth of that for non-liquefied layer.

Now, the above basic differential equation can be written in finite difference form as given by Poulos and Davis,1980 for a typical point i as (see figure 3.5).

$$EI\ [w(i-2) - 4w(i-1) + 6w(i) - 4w(i+1) + w(i+2)] / \delta^4 + k_i\ d.w_i = 0$$

$$\text{or,}\quad w(i-2) - 4w(i-1) + a_i\ w(i) - 4w(i+1) + w(i+2) = 0$$

$$\text{where}\quad a_i = 6 + (\ k_i\ l^4\ d\ /\ EI\ n^4\ )$$

where n = number of intervals along pile

ki = modulus of subgrade reaction k_h at ith node

δ = length of each subdivision along the pile

l = embedded length of the pile.

The above equation can be applied from point 2 to n to give n-1 equations.

Four further equations are obtained from the boundary conditions at the top and the tip of the pile.

For free head pile the boundary condition at the top is

$$\text{Shear} = EI\ \frac{d^3w}{dx^3} = H$$

$$\text{or}\quad w(-2) + 2w(-1) - 2w_2 + w_3 = \frac{H\ l^3}{EI\ n^3} \qquad (3.2.1)$$

where H is the horizontal laod at top end of the pile.

$$\text{Moment,}\quad EI\ \frac{d^2w}{dx^2} = H.e$$

$$w(2) - 2w(1) + w(-1) = \frac{H.el^2}{EI\ .n^2} \qquad (3.2.2)$$

where e is the distance of the lateral load at the top of the

Similarly for fixed head pile the rotation is zero at the top .

Thus $dw/dx = 0$

i.e. $w(2) - w(-1) = 0$ (3.2.3)

Now, at the lower tip of the pile treating it to be a free end

$$\text{Shear} = EI \frac{d^3w}{dx^3} = 0$$

i.e. $-w(n-1) + 2w(n) - 2w(n+2) + w(n+3) = 0$ (3.2.4)

$$\text{and moment} = EI \frac{d^2w}{dx^2} = 0$$

i.e. $w(n) - 2w(n+1) + w(n+2) = 0$ (3.2.5)

The final two equations are found from the overall equilibrium of the whole pile.

For horizontal force equilibrium i.e. $\Sigma F_H = 0$

As shown in the figure 3.5 the soil reaction at any node i is given by $Q_i = k_i \times A \times w(i)$

$= k_i \times (\delta \times d) \times w(i)$

Thus $\Sigma F_H = 0 \implies$

$H = \Sigma Q_i$

or $H = k_i (\delta \times d) \times w(i);\ i = 1, n+1$ (3.2.6)

or $H = k_1 (\delta \times d)/2 \cdot w(1) + k_2 (\delta \times d)\ w(2) + \ldots\ldots k_{n+1} (\delta \times d)/2\ (w_{(n+1)})$

Now for moment equalibrium of the whole structure i.e.

$\Sigma M = 0$

Taking moment of all the forces about the lower end of the pile.

$$H(e+1) = Q_1 x L + Q_2 (L-\delta) + Q_3 (L-2\delta) + \ldots\ldots Q_n (\delta) + Q_{n+1} \cdot (0)$$

it can also be written as

$$H(e+1)=k_1 \left(\frac{\delta x d}{2}\right) 1.w(1) + k_2(\delta x d)(1-\delta).w(2) + k_3(\delta x d)(1-2\delta) + w(3) + \ldots\ldots +k_n (\delta x d) (\delta).w_n \quad .(3.2.7)$$

Thus equations (1), (2), (4), (5), (6), (7) together with n-1 equations, apply for a free head pile and equations 1, 3, 4, 5, 6, 7 together with earlier (n-1) equations apply for fixed head pile.

Thus, having n+5 equations in n+5 unknowns, these simultaneous equations can be solved by using Gauss elemination technique. The method is available in any standard textbook (A.Ralston ,1965) on numerical methods of analysis.

3.2.1 Non-Dimensional Form of Solution:

The governing differential equation for pile can be written in a non-dimensional form and the flexural behaviour of the pile can be expressed in terms of non-dimensional parameters (Matlock and Reese,1960)

$$EI \frac{d^4 w}{dx^4} + k_h.d.w = 0$$

$$\frac{d^4 w}{dx^4} + \frac{k_h . d}{EI} w = 0$$

Putting $K = k_h.d$ and defining a term relative stiffness as $T = (EI/K)^{1/4}$,which has a dimension of length.Thus the differential equation becomes $(d^4w/dx^4 + T^4 \, . \, w = 0)$

Defining non-dimensional terms as follows:

Depth coefficient $Z = x/T$

Maximum depth coefficient $Z_{max} = L/T$

Soil Modulus function $r = KT^4/EI$

Deflection Coefficient $Ay = EI/H.T^3 \cdot w$

where H = Lateral Load at pile top.

The differential equation can be written as

$$d^4Ay/dZ^4 + r \cdot Ay = 0 \tag{3.2.8}$$

The corresponding boundary conditions can be written as follows:

<u>At Top End of Pile</u>

Shear : $$EI \frac{d^3w}{dx^3} = H$$

Putting $w = \frac{H.T^3}{EI} \cdot Ay$ and $Z = x/T$

we get, $$\frac{d^3Ay}{dZ^3} = 1 \tag{3.2.9}$$

Moment : $$EI \frac{d^2w}{dx^2} = 0 \tag{3.2.10}$$

we get, $$\frac{d^2Ay}{d\,Z^2} = 0$$

<u>At the bottom end of pile</u>

Shear : $$d^3Ay/dz^3 = 0 \tag{3.2.11}$$

Moment : $$d^2Ay/dz^2 = 0 \tag{3.2.12}$$

The equations (3.2.8), (3.2.9), (3.2.10), (3.2.11) and (3.2.12) can be written in the finite difference form and solved by Gauss elimination technique in the same manner as done previously.

The non-dimensional bending moment coefficients Am is given

by $\quad Am = \dfrac{EI}{H \cdot T} \cdot \dfrac{d^2 y}{dx^2}$

Thus, flexural behaviour of pile can be studied in terms of non-dimensional coefficients Ay and Am.

Pressure Variation
(free head short pile)

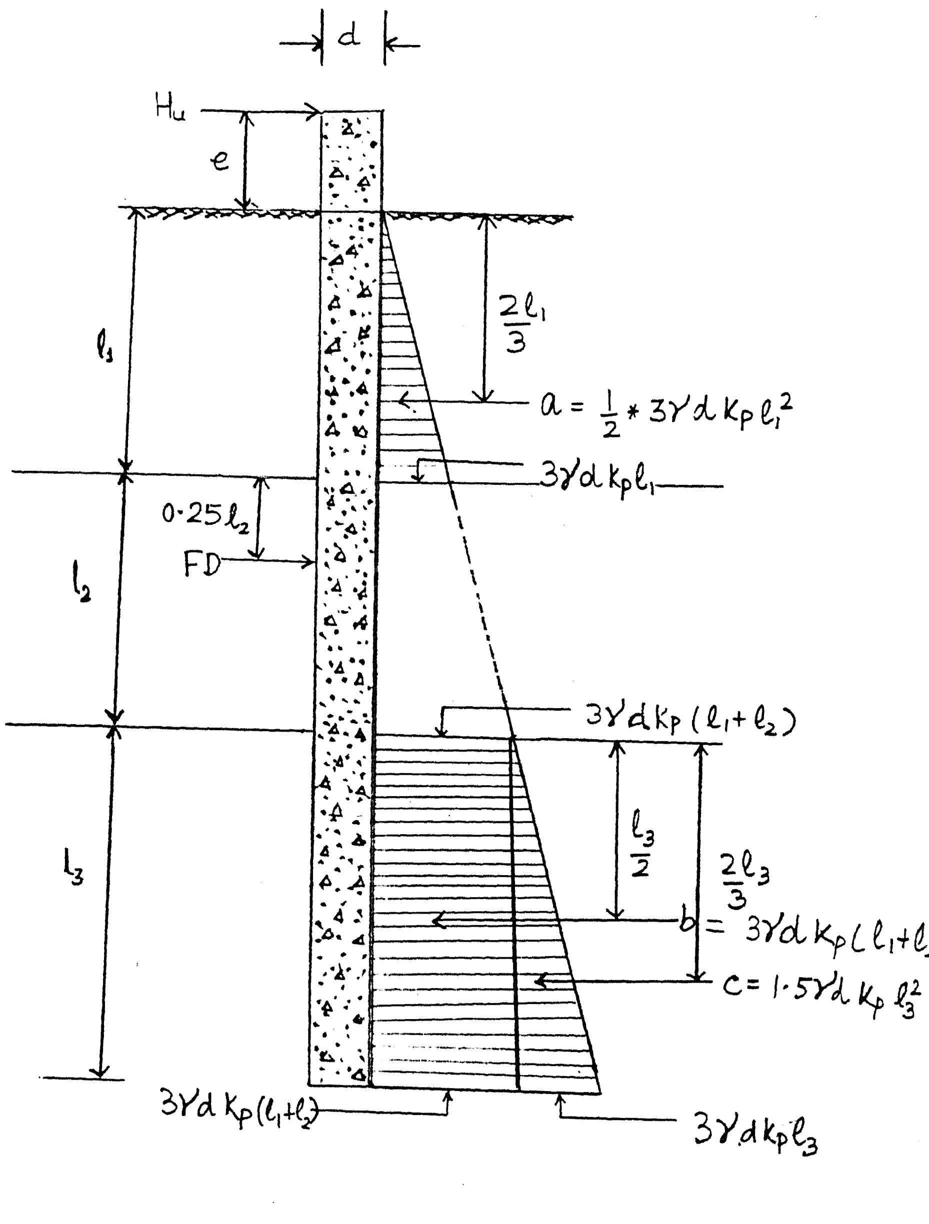


FIG.3.1

Pressure Variation (free head long pile)

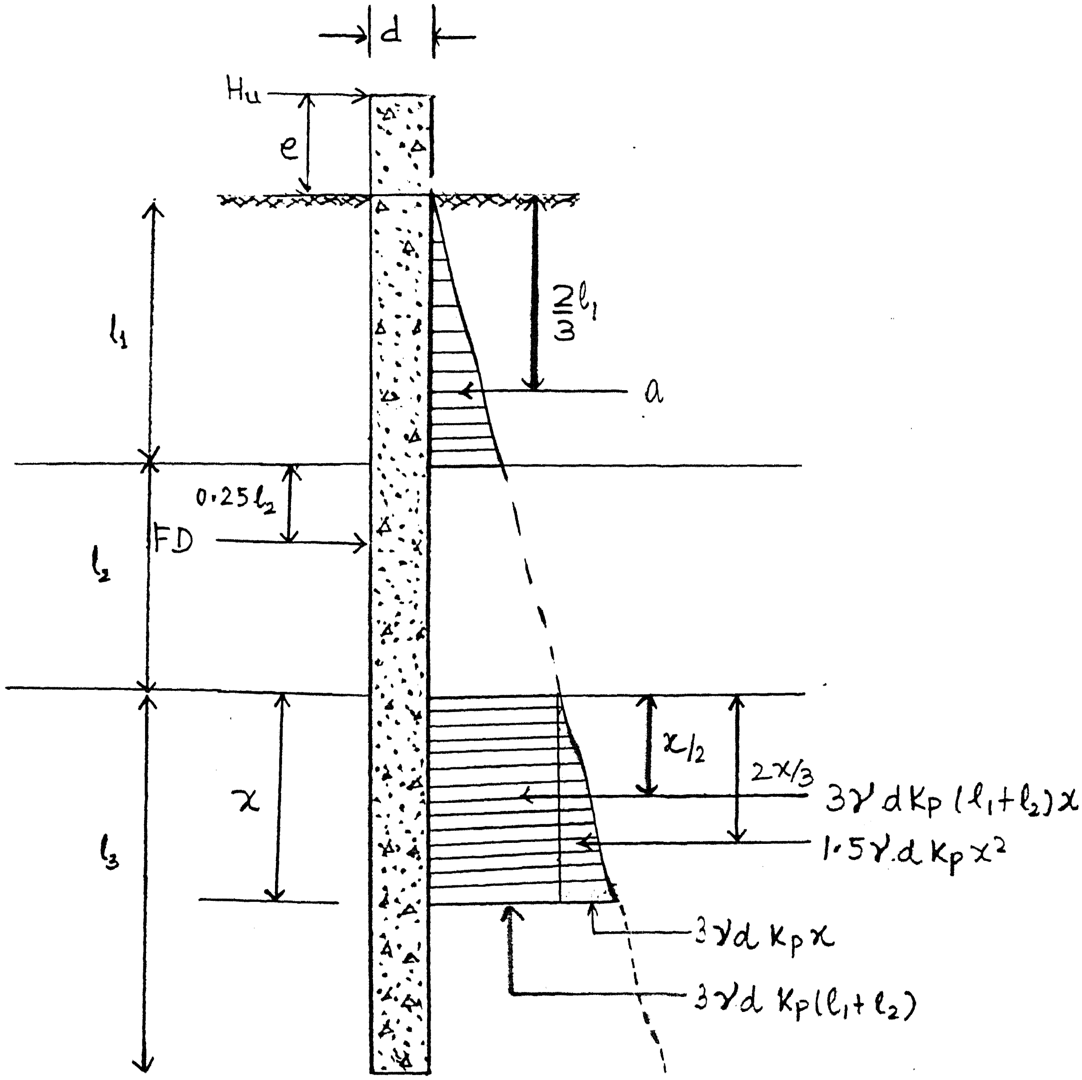


FIG.3.2

Pressure Variation (fixed head short pile)

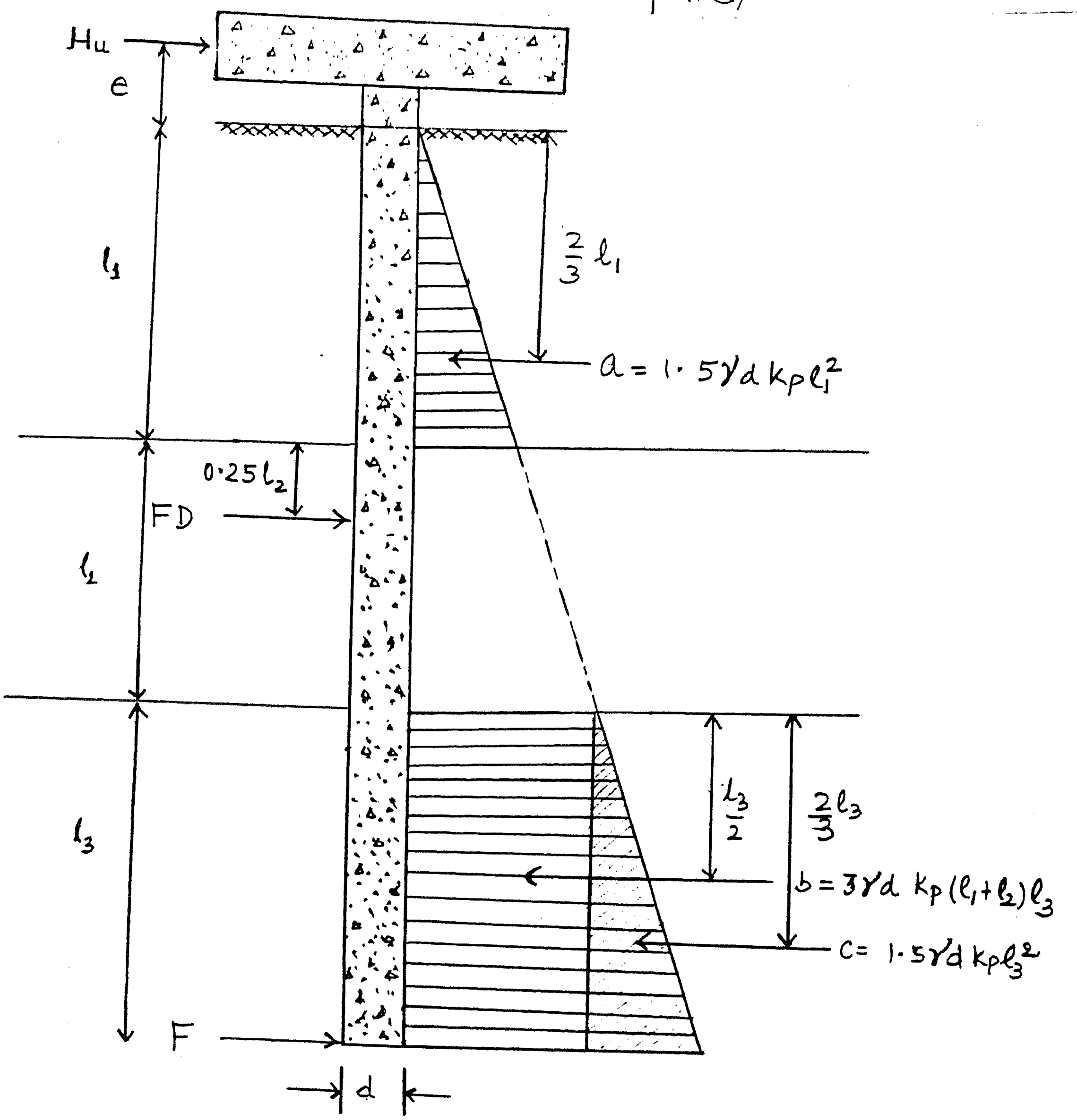


FIG. 3.3

Pressure Variation
(fixed head long pile)

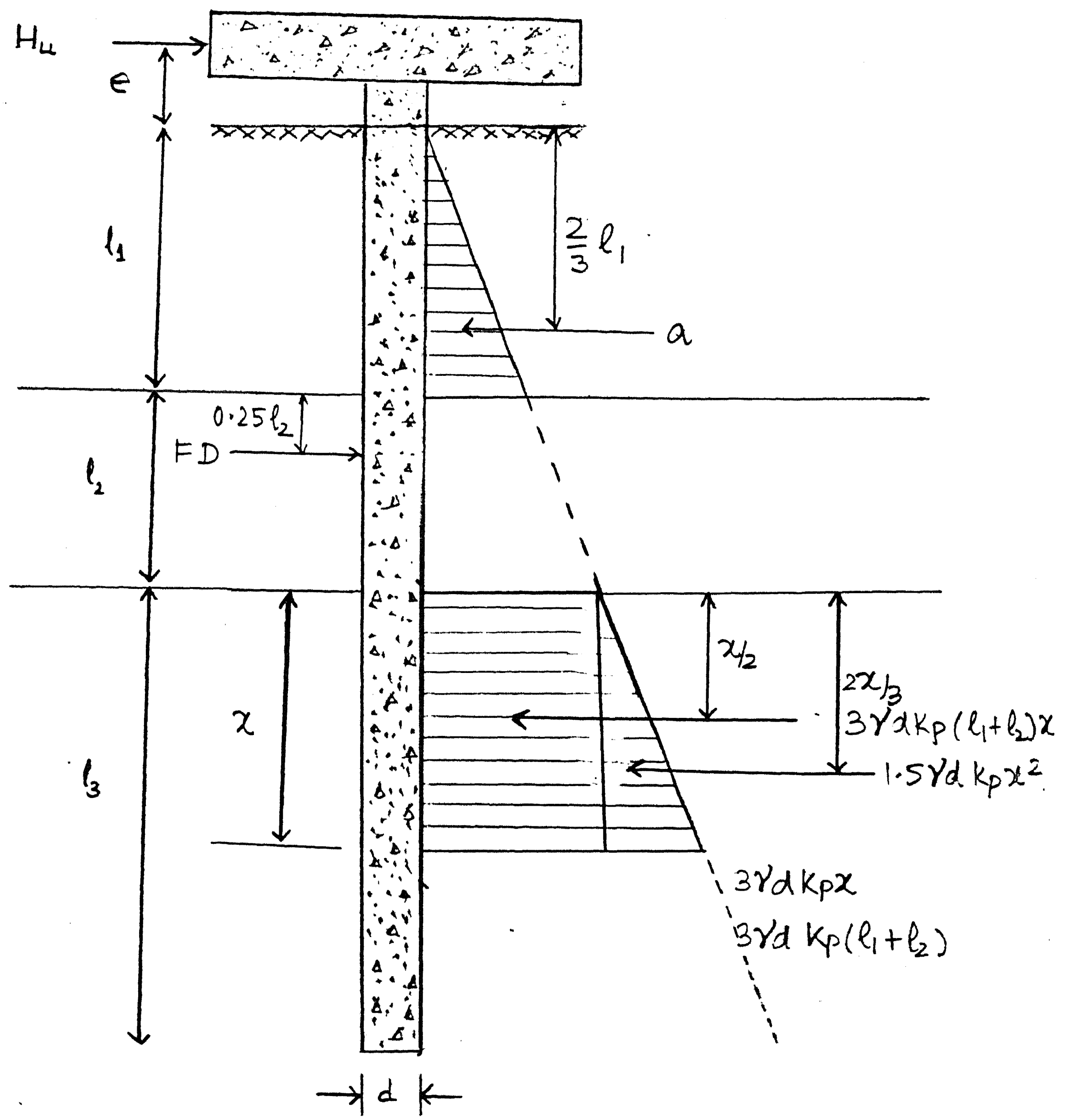


FIG.3.4

Points along the pile

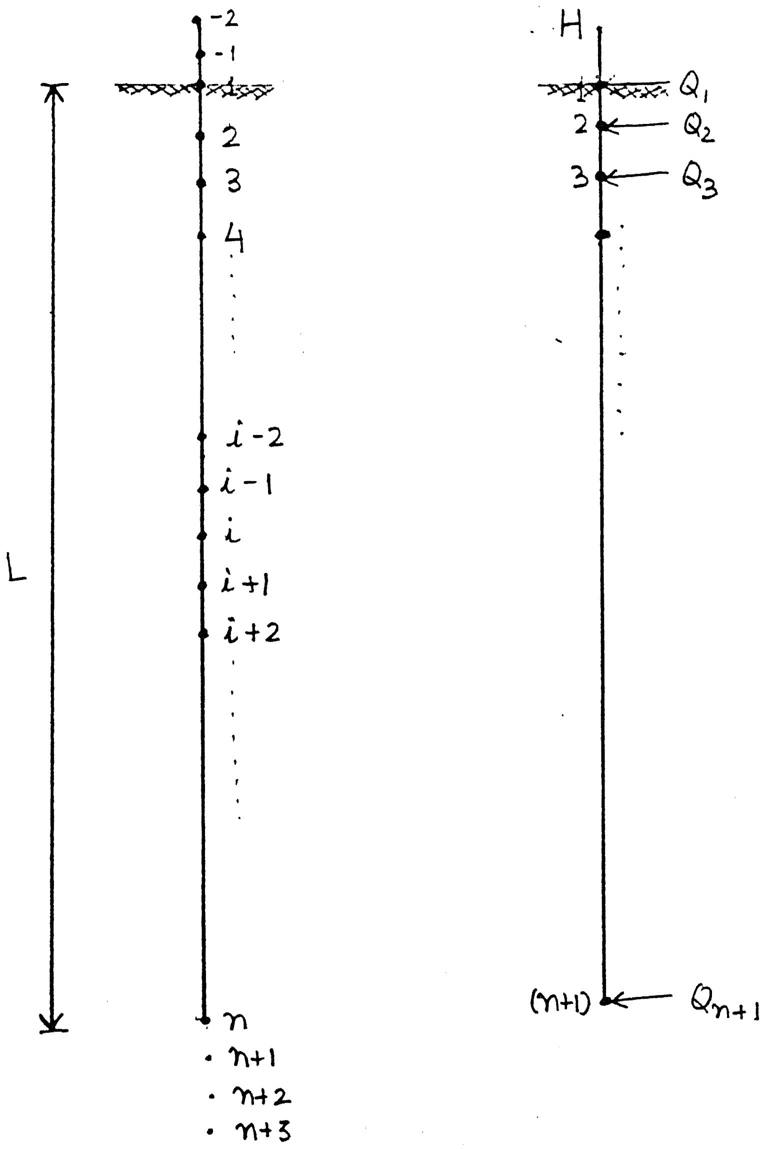


FIG.3.5.

4. RESULTS AND DISCUSSION

Based on the formulation prescribed in Section 3.1 for the ultimate lateral resistance of pile when certain depth of soil along the pile has liquefied, results have been provided in the forms of non-dimensional design charts for different cases of free head and fixed head piles.

Representative results have also been presented to highlight the effect of soil liquefaction on the flexural behaviour of laterally loaded piles, based on the analysis as discussed in Section 3.2. Computations have been made using HP9000 Computer System. The results and discussion pertaining to the lateral load capacity and flexural behaviour of piles are discussed separately as follows:

4.1 Lateral Load Capacity: Figures 4.1 to 4.10 show the ultimate lateral load capacity of piles in terms of non-dimensional parameters which are defined as follows:

$H_u/\gamma d K_p l^2$	:	Non-dimensional lateral load capacity (NLLC)
$M_u/\gamma d K_p . l^3$	:	Non-dimensional yield capacity of pile section (NYC)
l_2/l	:	Ratio of depth of liquefied zone to the total embedded length of the pile (NLD)
l_1/l	:	Ratio of depth of upper soil cover to the total embedded length of the pile (NCD).
e/l	:	Overhang ratio with respect to the embedded length of the pile (OHR)

In Figures 4.1, 4.2 and 4.3 the obtained results for the case of a free head short pile are presented.

It can be seen from the figure 4.1 that when OHR and NLD

are zero the well known Brom's solution for lateral load capacity i.e. NLLC = 0.5 is obtained. The figures indicate a non-linear relationship between NLLC and NLD. Curves marked 1, 2 and 3 in the figures 4.1, 4.2 and 4.3 show that as NLD increase NLLC decreases initially at an increasing rate, then at constant rate and finally at a decreasing rate. But curves marked 4 indicate continuous decrease in NLLC with NLD initially at a constant rate and then at a decreasing rate. These curves further indicate that in general for NLD value remaining constant as NCD increases NLLC decreases. This can be explained as follows:

The increasing value of l_1/l ratio (NCD) keeping l_2/l ratio (NLD) constant indicates that the liquefaction is initiated increasingly at a greater depth and extends farther. As the greater part of soil resistance is mobilised at greater depths, this causes significant loss of soil resistance over the lower portion of the pile length resulting in lower values of pile capacity (NLLC). But for the curve marked 4,corresponding to NCD equal to 0.4, the predicted values show the starting of a reverse trend beyond NLD equal to 0.4. It can be seen that for any fixed value of NLD (0.6 > NLD > 0.4) as NCD increases upto 0.2 NLLC decreases but with further decrease in NCD, NLLC increases.

The quantitative influence of l_1/l (NCD), l_2/l (NLD) on lateral capacity (NLLC) is estimated in terms of a ratio R which is defined as follows:

$$R = \frac{\text{NLLC for given values of NCD and NLD}}{\text{NLLC with NLD = 0}}$$

In Table 4.1 the values of this ratio (R) for different

NLD and NCD values are presented. To obtain the corresponding values of non-dimensional load capacity (NLLC) for OHR values of 0, 0.1 and 0.2 the values presented in the table should be multiplied by 0.5, 0.45 and 0.42 respectively. The table brings out quantitatively the effect of the depth of soil cover (NCD) the depth of liquefied zone (NLD) on the lateral load capacity over the entire range of NLD from 0 to 1.0, and NCD from 0 to 0.4.

In Figure 4.4 the obtained result for the case of fixed head short pile is presented. It can be seen from the figure that when l_2/l (NLD) is zero, the Brom's solution for lateral load capacity for non-liquefied case i.e. NLLC = 1.5 is obtained. Here also a non-linear relationship of NLLC and NLD is indicated. The NLLC values decrease at an increasing rate with increasing values of NLD. It is seen from the figure that for a given value of NLD, the NLLC decreases with the increasing value of NCD. The explanation of this behaviour is same as that of free head short pile case.

Table 4.2 brings out the effect of soil liquefaction and soil cover on the lateral load capacity of piles quantitatively.

In figures 4.5, 4.6 and 4.7 the variation of the non dimensional lateral load capacity (NLLC) with non -dimensional yield capacity (NYC) for the free head long pile case with different overhang ratio of 0, 0.1 and 0.2 are presented. These design charts have been drawn for the case when the cover ratio (NCD) is zero, and the liquefaction starts right from the ground surface. Thus the NLD=0, case in the present study (curve makred 1) is the same as that of Brom's (1964) non-

liquefied case. The general trend of these curves indicate the following:

- For a given OHR and NYC as NLD increases, NLLC decreases.
- For a given OHR and NLD as NYC increases NLLC also increases at increasing rates.

With the help of Table 4.3, for free head long piles a quantitative estimation of the influence of various parameters can be made. It is clear from the Table that for a increase in NLD from 0 to 0.6, there is a decrease in the load capacity of the pile ranges from 30 to 73 percent for different moment capacities of the pile. It is also seen from the table that for a variation in yield capacity from 0.013 to 0.42 (32 fold increase) the corresponding increase in load capacity is in the range of 10.3 to 19 times.

In figures 4.8, 4.9 and 4.10 variation of NLLC with NYC for the fixed head long pile case, with different overhang ratios of 0, 0.1 and 0.2 are shown. The general trend of the presented curves is similar to that of the free head long pile case viz.

- For a given OHR and NYC, NLLC decreases with increasing NLD.
- NLLC increases at an increasing rates with NYC for a given OHR and NLD.

Table 4.4 gives a quantitative estimation of the influence of the various parameters on ultimate lateral load capacity for fixed head piles. It is seen from the table that for a 32 fold increase in NYC (from 0.013 to 0.42) the load capacity (NLLC) increase in the range of 8.13 to 29 times. For an increase in NLD from 0.0 to 0.6 the NYC decrease by about 20-66 percent.

An illustrative example for the studied range of values of NYC an

OHR has been presented as follows to show the use of the develope non-dimensional design charts.

Example :

Given :

Yield moment capacity of Pile section Mu =500 kNm

Unit Weight of soil (effective) = 6 kN/m^3

Cover depth, $l_1 = 0$,

Total embedded length of pile, l = 10 m.

Passive earth pressure coefficient of soil, Kp=3

Diameter of Pile, d = 0.5 m.

Find : Ultimate Lateral Load Capacity of both free head and fixed head piles when

Case A : (i) Depth of soil liquefaction l_2 = 4.0 m

(ii) There is no liquefaction, $l_2 = 0$, and overhang length e = 0.0 and 1.0 m.

Case B : Depth of soil liquefaction l_2 = 4.0 and the overhang length e = 0.7 m.

Solution:-

From the given data :

$$\frac{M_u}{\gamma dKpl^3} = \frac{500}{10x0.5x3x10^3} = 0.053$$

$$\frac{l_2}{l} = \frac{4}{10} = 0.4$$

<u>Free Head Pile</u> : From Figure 4.5 for $\frac{Mu}{\gamma dKpl^3} = 0.053$

and $\frac{l_2}{l} = 0.4$

when $\dfrac{e}{l} = 0.0$ we get $\dfrac{H_u}{\gamma d K_p l^2} = 0.13$

$====> Hu = 117$ kN

When $e/l = 0.1$, from figure 4.6, $\dfrac{H_u}{\gamma d K p l^2} = 0.11$

$======> Hu = 99$ kN

For no liquefaction case i.e. when $l_2/l = 0.0$

For $e/l = 0.0$, from figure 4.5

$$\frac{H_u}{\gamma d K p l^2} = 0.25$$

$=====> Hu = 225$ kN

For $e/l = 0.1$ from Figure 4.6

$$\frac{H_u}{\gamma d K p l^2} = 0.19$$

$=====> H_u = 171$ kN

<u>Fixed Head File</u> :- From figure 4.8 ($e/l = 0.0$) for $l_2/l = 0.4$

we get $\dfrac{H_u}{\gamma d K_p l^2} = 0.25$

$=====> H_u = 225$ kN

From figure 4.9 ($e/l = 0.1$) for $l_2/l = 0.4$

we get $\dfrac{H_u}{\gamma d K_p l^2} = 0.21$

$=====> H_u = 190$ kN

For no liquefaction case.

For $e/l = 0.0$ from figure 4.8

we get $$\frac{H_u}{\gamma d K_p l^2} = 0.37$$

$\Longrightarrow$ H_u = 333 kN

For e/l = 0.1, from figure 4.9

we get $$\frac{H_u}{\gamma\, d K_p l^2} = 0.30$$

$\Longrightarrow$ H_u = 270 kN

Case B :

For intermediate values of e/l the value of Hu can be calculated by linear interpolation.e.g. for data given above to be the same if e/l = 0.07 then Hu can be calculated as follows:

For free head case

H_u = 117 kN, when e/l = 0.0, l_2/l = 0.4, $\frac{Mu}{\gamma d K_p l^3} = 0.053$

H_u = 99 kN, when e/l = 0.1,

then for e/l = 0.07,

$$H_u = 117 - \frac{117-99}{0.1-0.0} \times 0.07$$

= 104.4 kN

Similarly for fixed head pile

H_u = 225 kN, e/l = 0.0

H_u = 190 kN, e/l = 0.1

For e/l = 0.07,

$$H_u = 225 - \frac{225-190}{0.1-0.0} \times 0.07$$

= 200.5 kN

4.2 Flexural Behaviour

As already stated earlier the governing differential equation to find the flexural response of a pile in a non-dimensional form is:

$$\frac{d^4Ay}{dZ^4} + r.Ay = 0$$

The boundary conditions for free head and fixed head pile case has already been given in section 3.2. The flexural behaviour of pile has been provided in terms of non-dimensional depth,deflection and bending moment coefficients for various cases of pile length,

depth of liquefied zone and the soil cover over the liquefied soil. The non-dimensional terms used has been defined in section 3.2. The other terms used are as follows:

Cover depth coefficient : $Z_1 = L_1/T$

Liquefied depth coefficient : $Z_2 = L_2/T$

In the presented analysis over the depth Z_1, Z_2 and Z_3 the original average SPT values when there was no liquefaction are taken as N_1, k_2N_1 and k_3N_1 respectively. When liquefaction occurs, SPT value is reduced and in the present analysis it has been taken as $k_2N_1/10$ over the liquefied depth as suggested (Yoshida and Hamada,1991). For computational purpose k_2 and k_3 has been taken as 0.5 and 1.25 respectively. With these values the value of r in different zones would be 1.0, 0.05 and 1.25.

The flexural behaviour of pile has been discussed in figures 4.11 to 4.26 for different values of cover depth Z_1, depth of liquefaction Z_2, pile length Z_m. The different figures have been discussed separately as follows:

Figures 4.11, 4.12 and 4.13 are for the free head pile case with $Z_m = 2.0$. For this value of Z_m pile behaves as a short pile. These three figures together show the effect of soil cover Z_1 on the flexural behaviour of pile when the typical depth $(Z_1 + Z_2)$ upto which liquefaction is considered to be possible is constant and is equal to 0.8. Figure 4.11 shows the deflection and bending moment variation along the length of the pile for the two cases when these is no liquefaction i.e. $Z_2 = 0$ (curve 2) and when there is liquefaction over $Z_2 = 0.4$ (curve 1). Curve 3 and Curve 4 are similarly for bending moment variation.Figures 4.12 and 4.13 show the variation of deflection and bending moment respectively along the length of the pile with Z_1 varying from 0.0 to 0.6. Based on the results presented in Figures 4.11, 4.12 and 4.13, the ratio of the maximum deflection (Ay) and moment (Am) of a pile for different values of Z_1 (soil cover), Z_2 (non-dimensional liquefied depth) with the corresponding values for the non-liquefied case are presented in Table 4.5. It is clear from the table that as the cover Z_1 decreases from 0.8 down to 0.0 the relative values of the deflection coefficient Ay at the top of the pile increases by 5.4 times. It is seen from the table that with the decrease in the depth of non-liquefied soil cover the value of maximum bending moment coefficient Am, decreases upto a certain extent but when the cover is absent the Am value increases to its maximum value which is 2.6 times the value when there was no soil liquefaction at all. The above behaviour can be explained as follows:

For a given depth $Z_1 + Z_2$ as the non-liquefied cover depth Z_1 decreases, the liquefied depth increases, the deflection of the pile also increases. This results in greater mobilization of soil resistance from the upper non-liquefied soil cover. This increased soil resistance in upper soil cover reduces the maximum bending moment in the pile at a lower section. But when the soil cover becomes very small, there is little resistance from the upper soil cover and there by having an insignificant influence on moment; so again the maximum bending moment coefficient value increases.

Figures 4.14, 4.15 (a), 4.15 (b) and 4.16 are also for the free head pile case but with $Z_m = 4.0$ for which the pile may be treated as a long pile. These three figures show the effect of soil cover Z_1 on the flexural behaviour of pile when the depth, (Z_1+Z_2) is considered constant and is equal to 1.6. Figures 4.15 and 4.16 show the variation of deflection and bending moment along the length of the pile respectively. The results of figure 4.14, 4.15(a), 4.15(b) and 4.16 are presented in table 4.6. It is clear from the table that as the depth of cover Z_1 decreases from 1.6 down to 0.0 the relative Ay value at the top of the pile increases by 5.95 times. The table shows that for this case also the maximum bending moment coefficient Am initially decreases with decreasing non-liquefied soil cover. But when the soil cover becomes very small the Am value starts increasing. For no soil cover the maximum Am value is 3.0 times than that for the case when there was no soil liquefaction. This behaviour can also be explained as above.

Figures 4.17 and 4.18 are for the free head pile. Here maximum depth coefficient is equal to $Z_m = 4.0$ which indicates

that the pile is a long pile. These two figures show the effect of depth of liquefaction Z_2 on the flexural behaviour of pile when depth of cover Z_1 is constant and is equal to 0.4. Figures 4.17 and 4.18 show respectively the deflection and bending moment variation along the pile length. The results of figures 4.17 and 4.18 are shown in Table 4.7. It is clear from the table that as Z2 varies from 0.0 to 1.6 the relative value of Ay at the top of pile increases in the ratio of 1 to 1.52. The table also shows that as the depth of liquefaction Z_2 increases for a constant cover depth Z_1, the maximum bending moment coefficient Am decreases. It reduces to 0.62 times the initial value corresponding to no liquefaction. This behaviour can also be explained similarly as above.

Figures 4.19, 4.20 and 4.21 are for the fixed head pile with $Z_m = 2.0$. Thus it is a short pile. These figures show the effect of cover Z_1 on the flexural behaviour of pile when (Z_1+Z_2) is constant and is equal to 0.8. The maximum deflection and bending moment occurs at the top of the pile. Table 4.8 shows the relative variation of Ay and Am at the top of the pile. When the cover Z_1 varies form 0.8 down to 0.0 the relative Ay value at top of the pile increases by 1.95 times. The table shows that for given $Z_1 + Z_2$ as the non-liquefied cover depth decreases the Am value first decreases and then starts increasing. The Am value for no soil cover case is 1.48 times the Am value for a cover of $Z_1 = 0.8$ i.e. when there is no liquefaction. This behaviour can be explained in a similar manner as that of the free head pile case. These variations are less as compared to the case of free head pile, thus showing the importance of the effect of fixity at the top of

the pile for the fixed head case.

Figures 4.22, 4.23 and 4.24 are for the fixed head pile with $Z_m = 4.0$. These figures show the effect of Z_1 on flexural behaviour of a long pile. Table 4.9 shows the relative variation in Ay and Am with Z_1. As Z_1 decreases from 1.6 to 0.0 the Ay value increases by about 3.7 times. For this case also, the maximum Am value first decreases with decreasing non-liquefied soil cover and finally starts increasing. Here the maximum Am value for no soil cover case is 2.2 times the Am value for the case of $Z_1 = 1.6$ i.e. non-liquefied case.

Figures 4.25 and 4.26 show the effect of depth of liquefaction on the flexural behaviour of a fixed head long pile (i.e. $Z_m = 4.0$). Here the constant cover provided is $Z_1 = 0.4$. It is clear from the table 4.10 that as Z_2 increases from 0.0 to 1.6, the relative Ay values at the top increases by 2 times and Am values increase by 1.07 times. The variation in maximum bending moment coefficient Am is very small as the depth of liquefaction increases from 0.0 to 1.6.

FREE HEAD-SHORT PILE
(NLD vs NLLC)

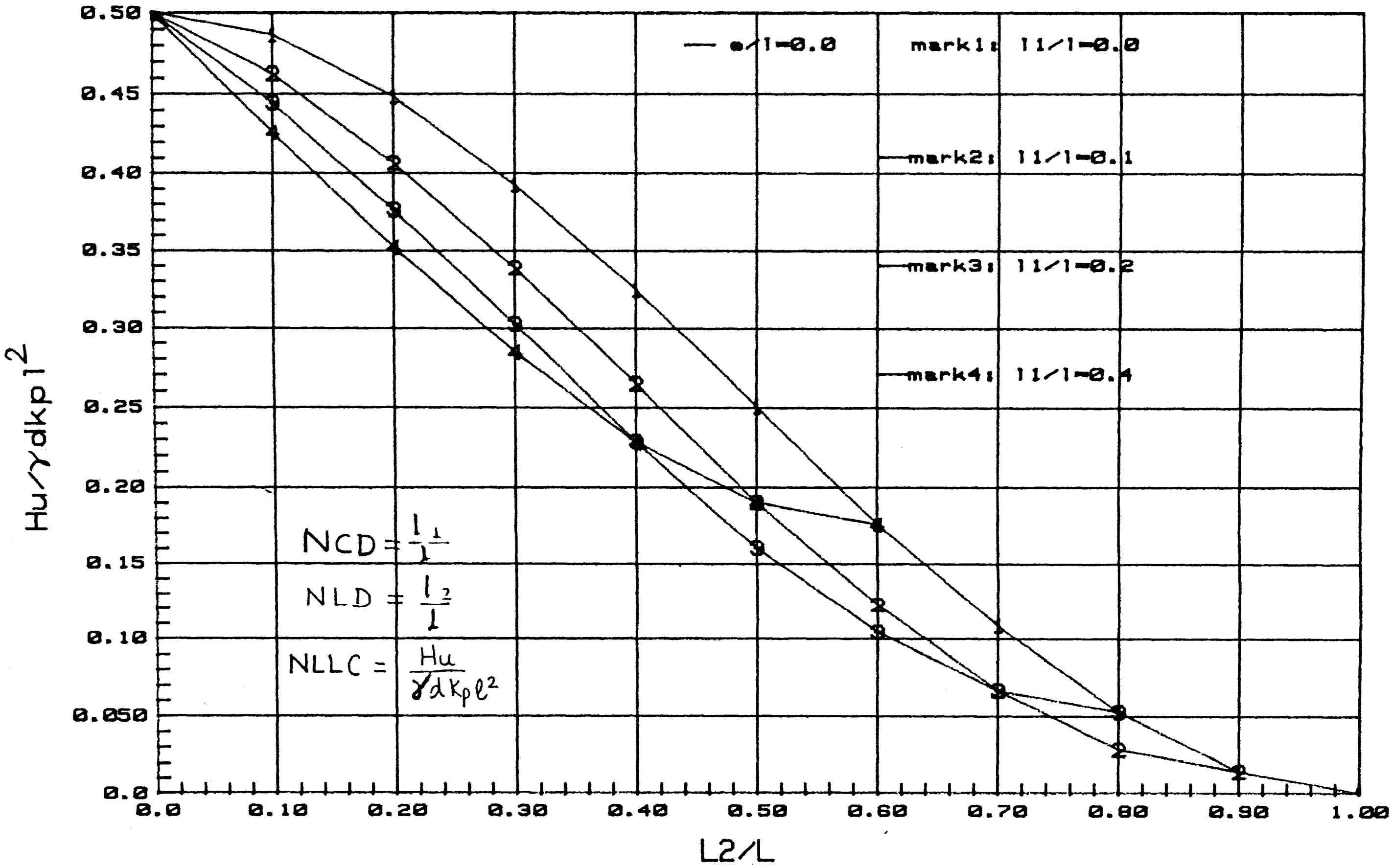


FIG. 4.1

FREE HEAD-SHORT PILE
(NLD vs NLLC)

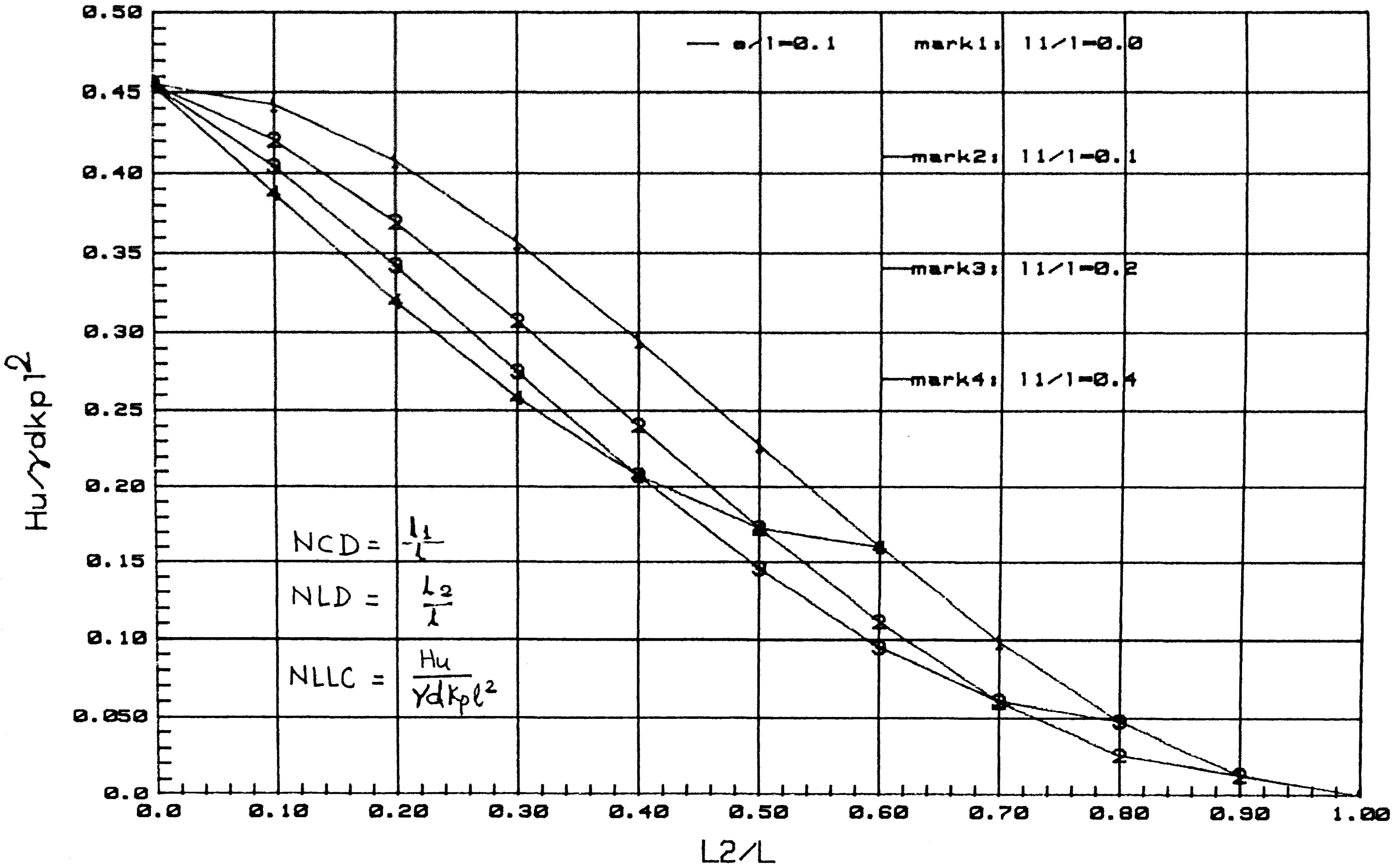


FIG. 4.2

FREE HEAD-SHORT PILE

(NLD vs. NLLC)

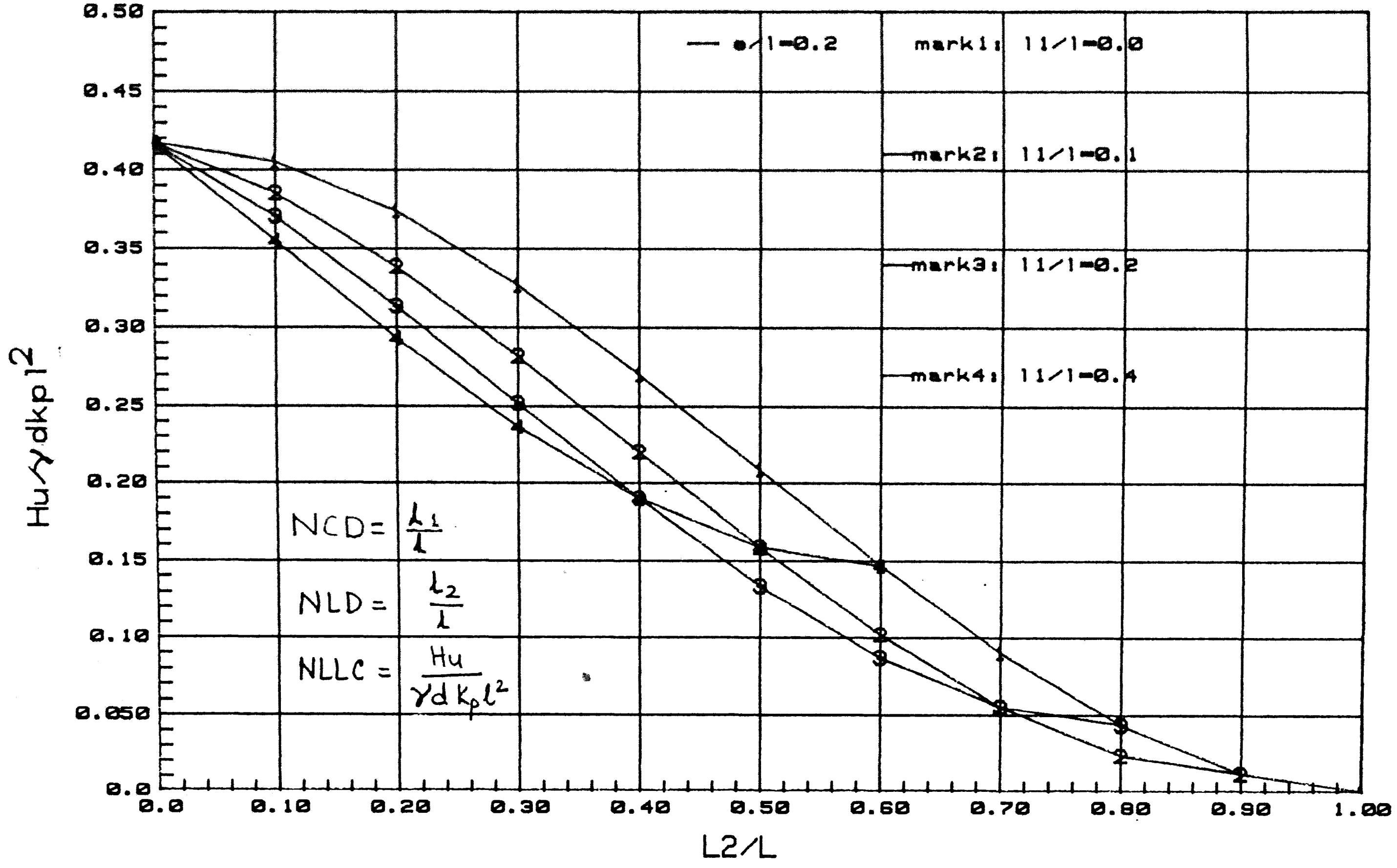


FIG.4.3

FIXED HEAD-SHORT PILE

(NLD vs NLLC)

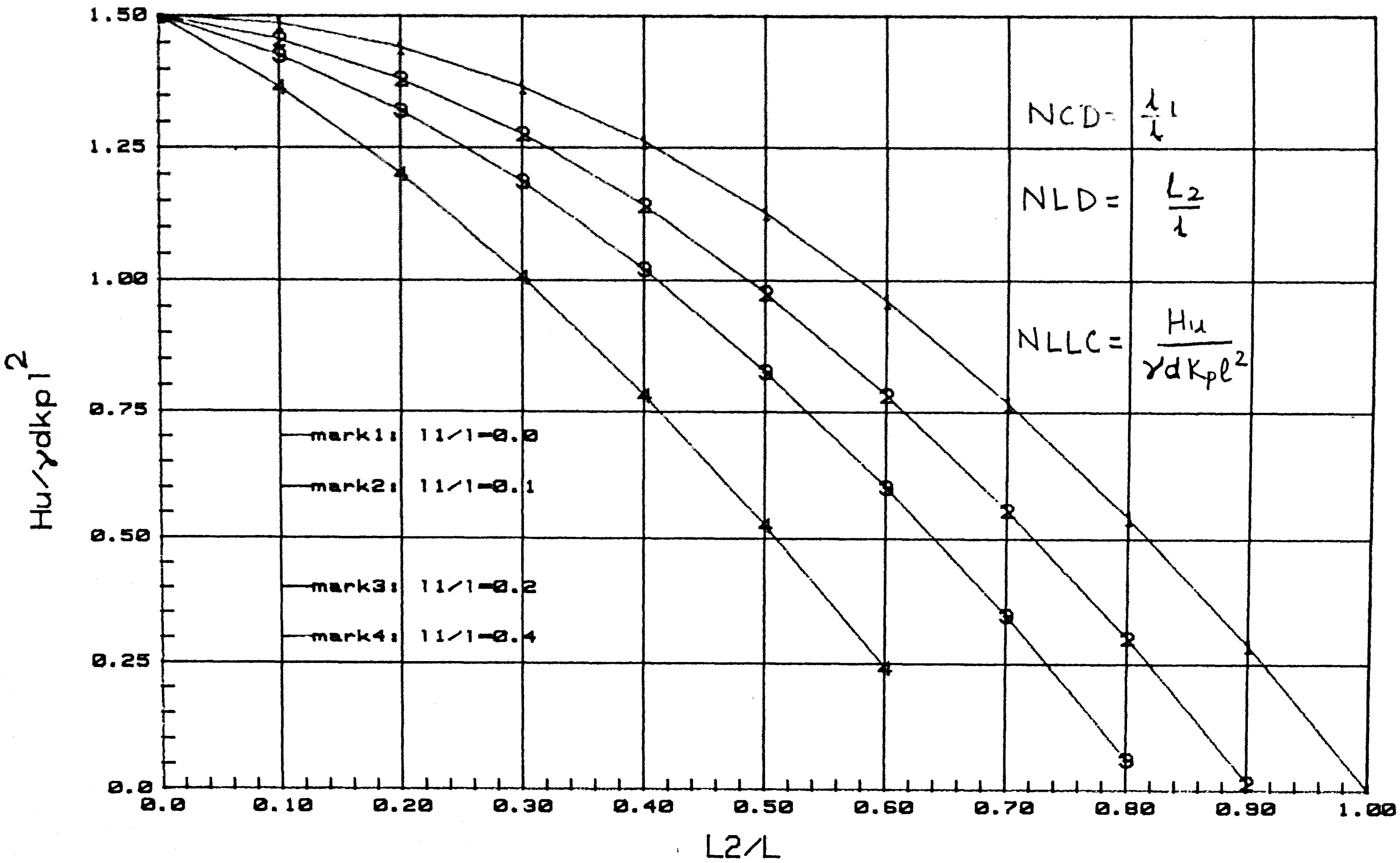


FIG.4.4

FREE HEAD -LONG PILE

(NYC vs NLLC)

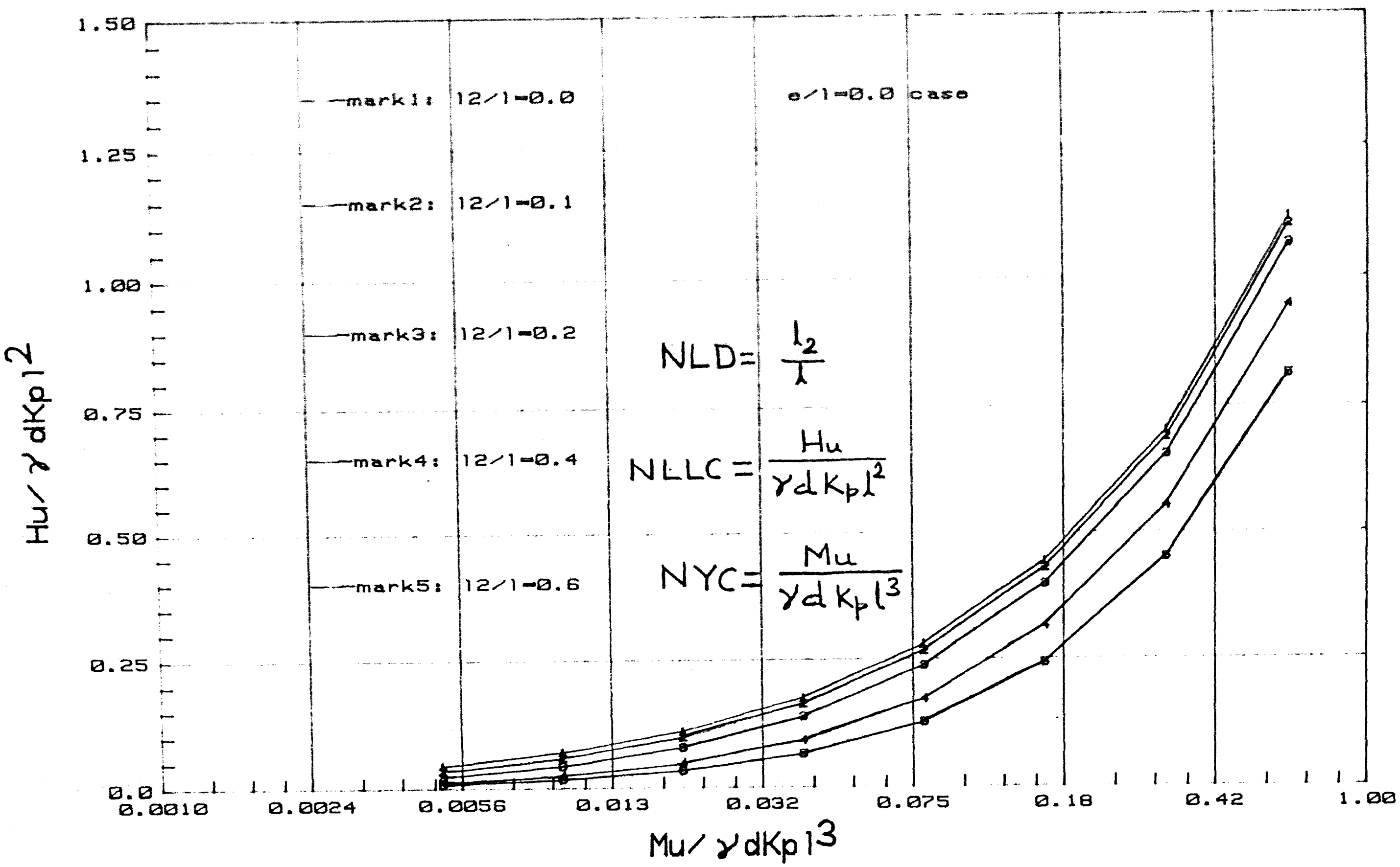


FIG. 4.5

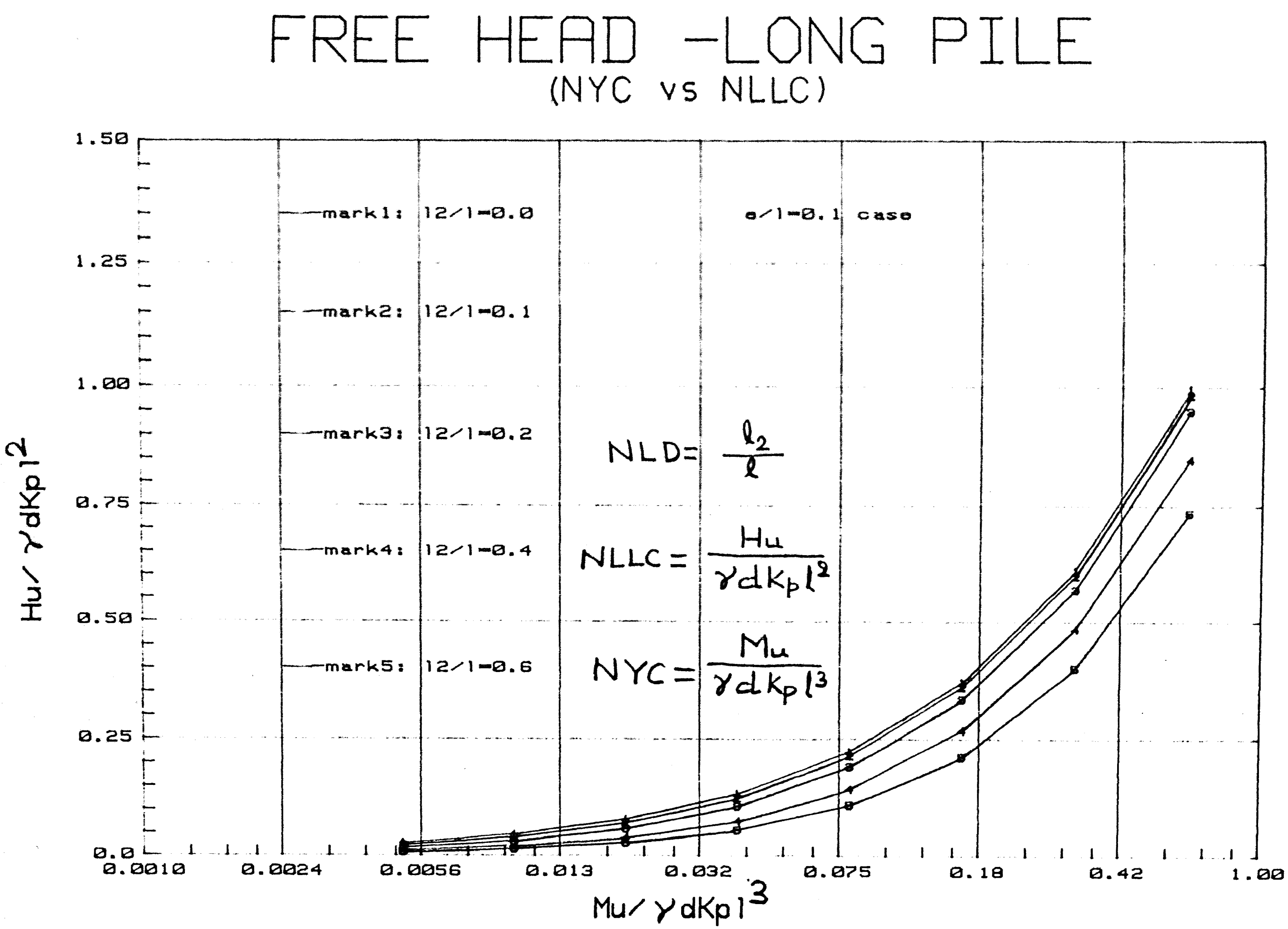

FREE HEAD -LONG PILE
(NYC vs NLLC)
mark1: 12/l=0.0
mark2: 12/l=0.1
mark3: 12/l=0.2
mark4: 12/l=0.4
mark5: 12/l=0.6
e/l=0.1 case
$NLD = \frac{l_2}{l}$
$NLLC = \frac{Hu}{\gamma d K_p l^2}$
$NYC = \frac{Mu}{\gamma d K_p l^3}$
1.50
1.25
1.00
0.75
0.50
0.25
0.0
0.0010
0.0024
0.0056
0.013
0.032
0.075
0.18
0.42
1.00
$Hu/\gamma dKpl^2$
$Mu/\gamma dKpl^3$

FIG.4.6

FREE HEAD -LONG PILE
(NYC vs NLLC)

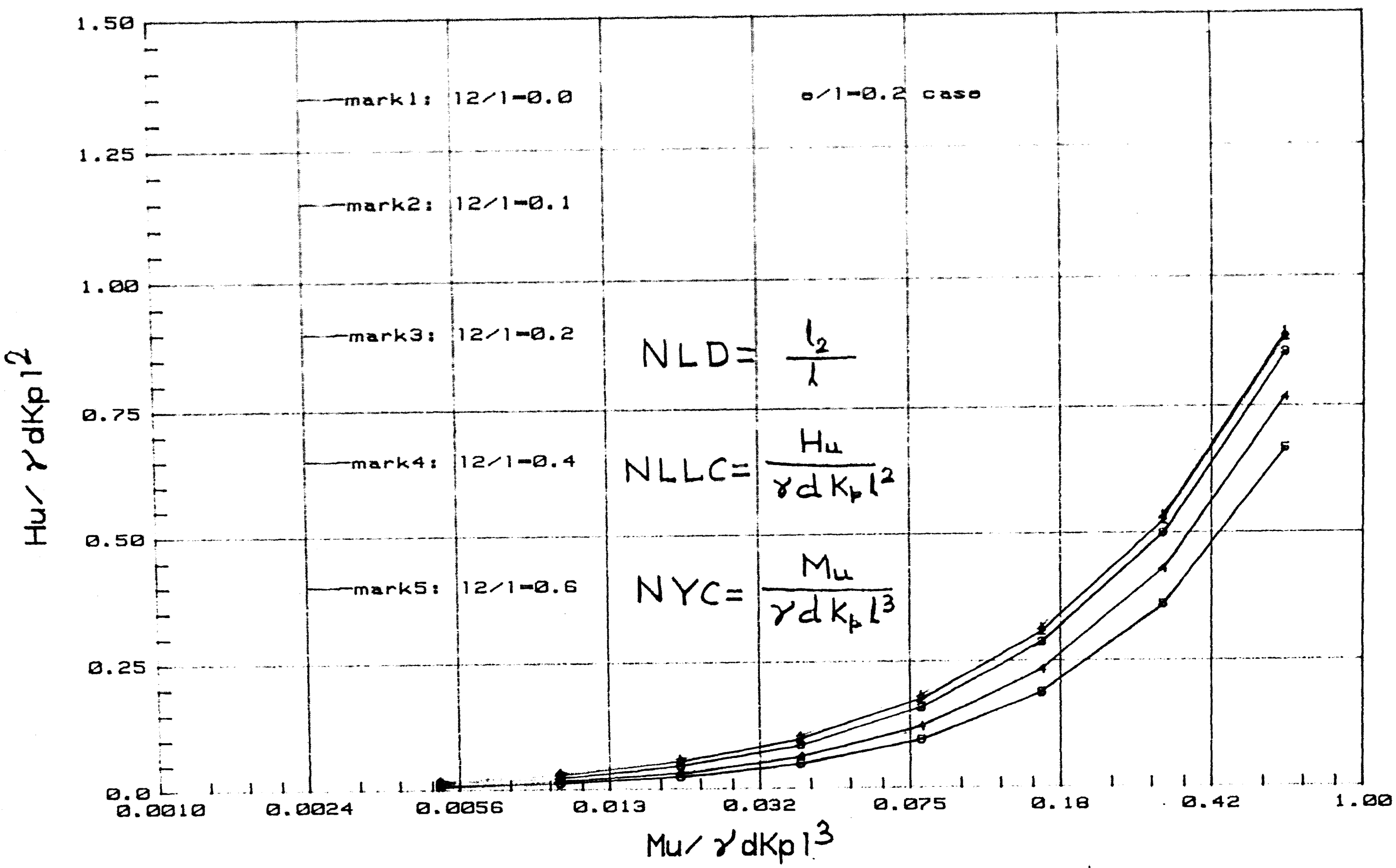
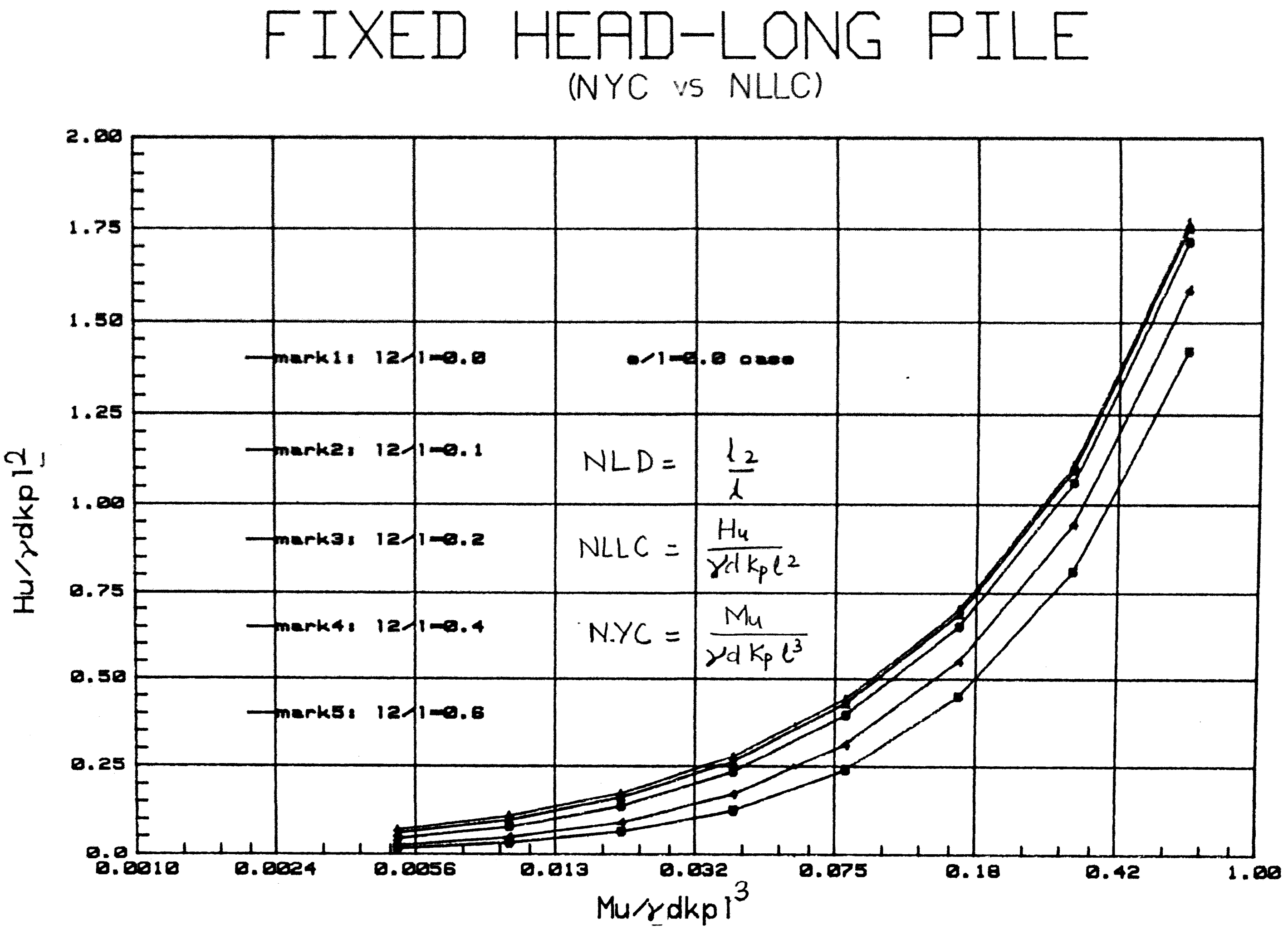


FIG.4.7



FIXED HEAD-LONG PILE
(NYC vs NLLC)
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.0
Hu/γdkpl2
0.0010
0.0024
0.0056
0.013
0.032
0.075
0.18
0.42
1.00
Mu/γdkpl3
mark1: l2/l=0.0
mark2: l2/l=0.1
mark3: l2/l=0.2
mark4: l2/l=0.4
mark5: l2/l=0.6
e/l=0.0 case
NLD = l2/l
NLLC = Hu/(γd Kp l2)
NYC = Mu/(γd Kp l3)

FIG.4.8

FIXED HEAD-LONG PILE
(NYC vs NLLC)

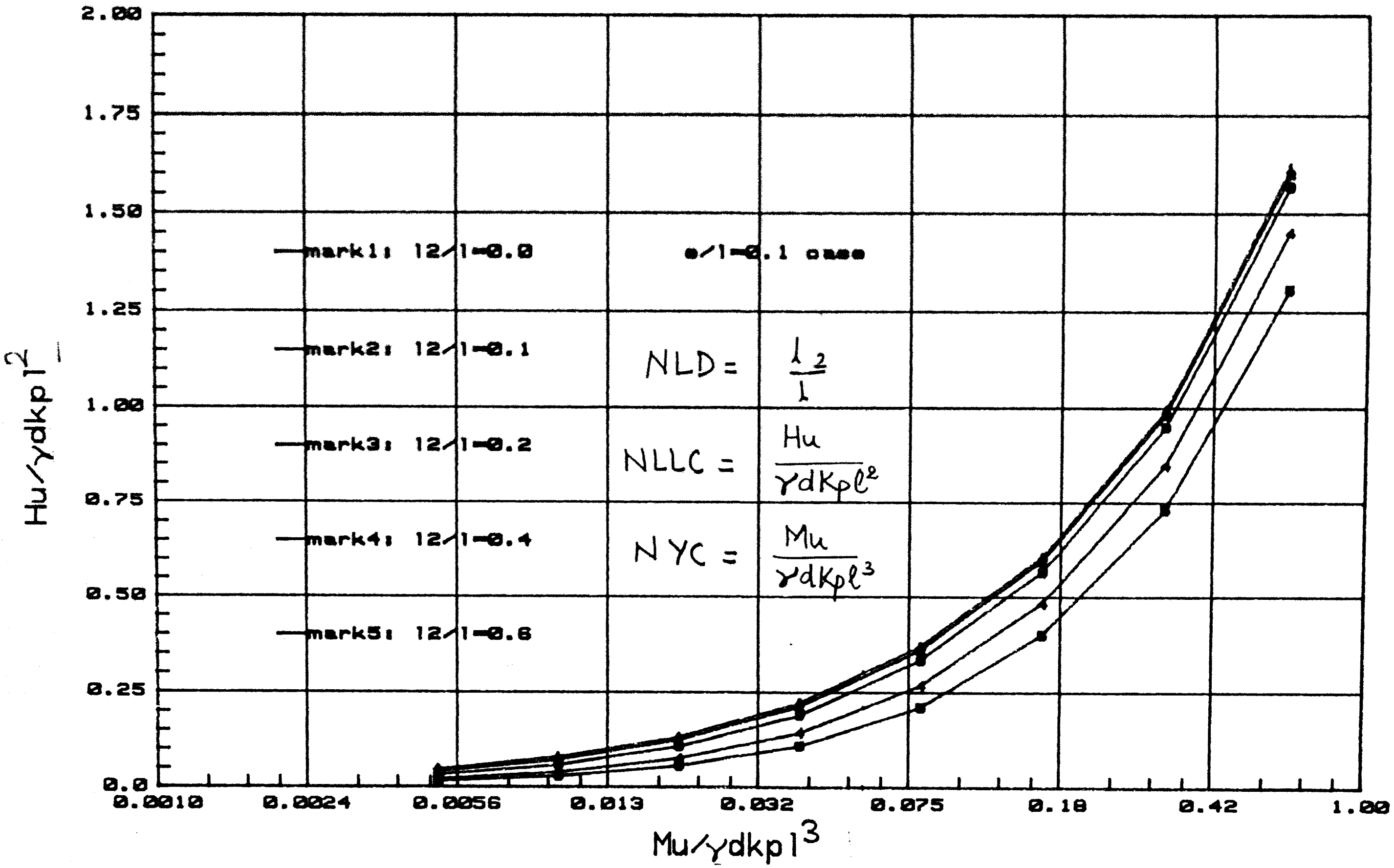


FIG.4.9

FIXED HEAD-LONG PILE

(NYC vs NLLC)

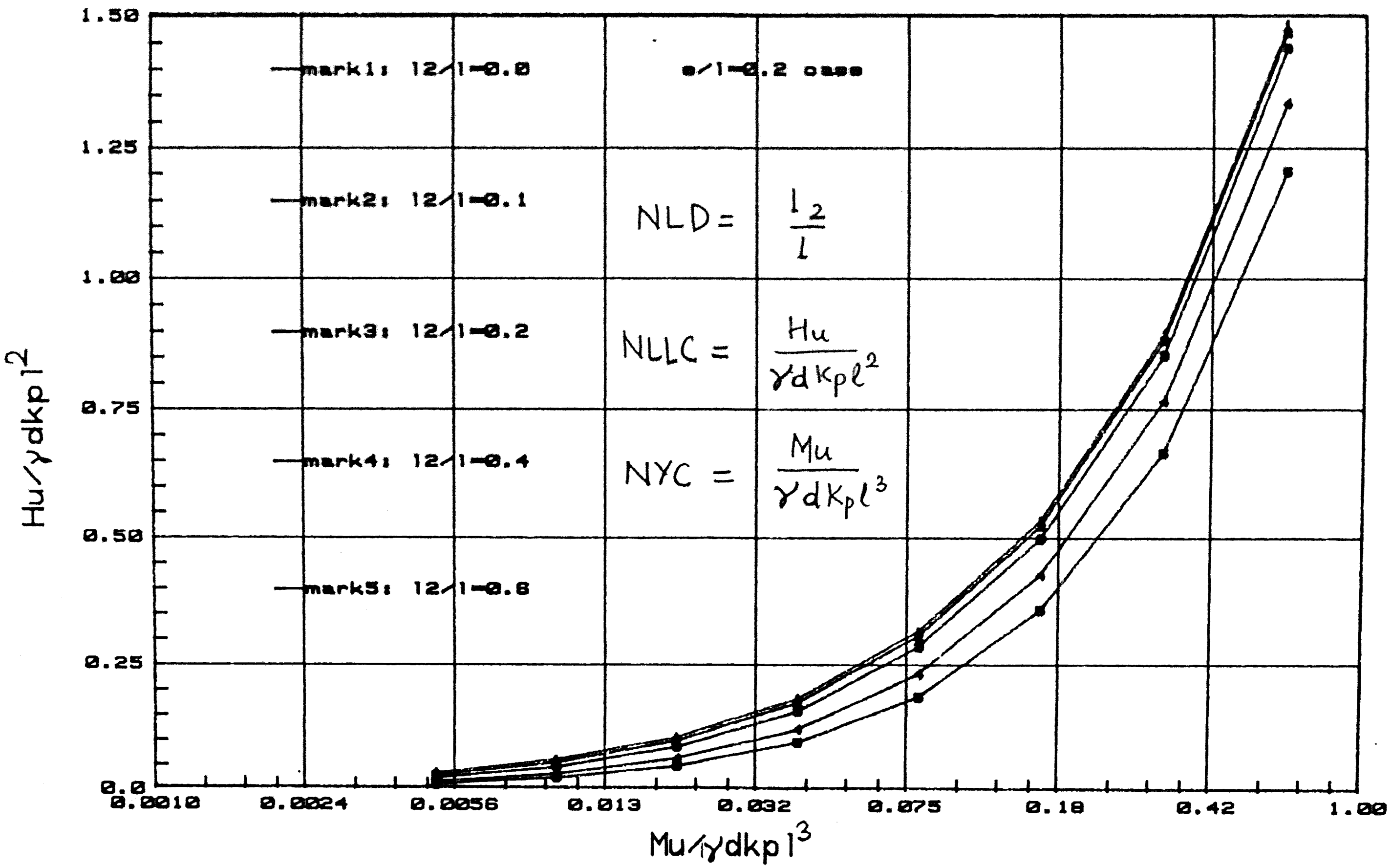


FIG.4.10

FREE HEAD PILE

(depth vs deflection, moment)

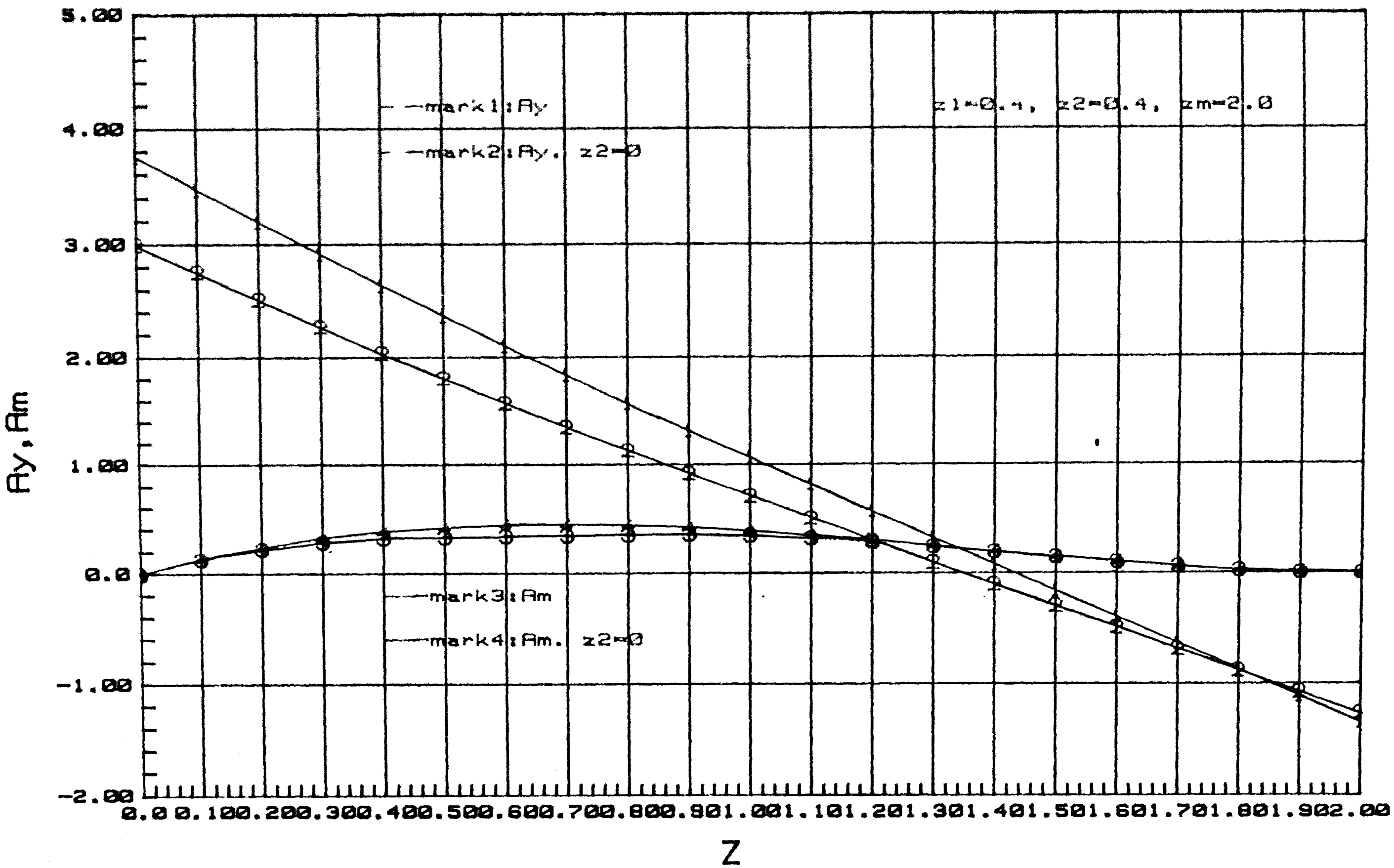


FIG 4.11

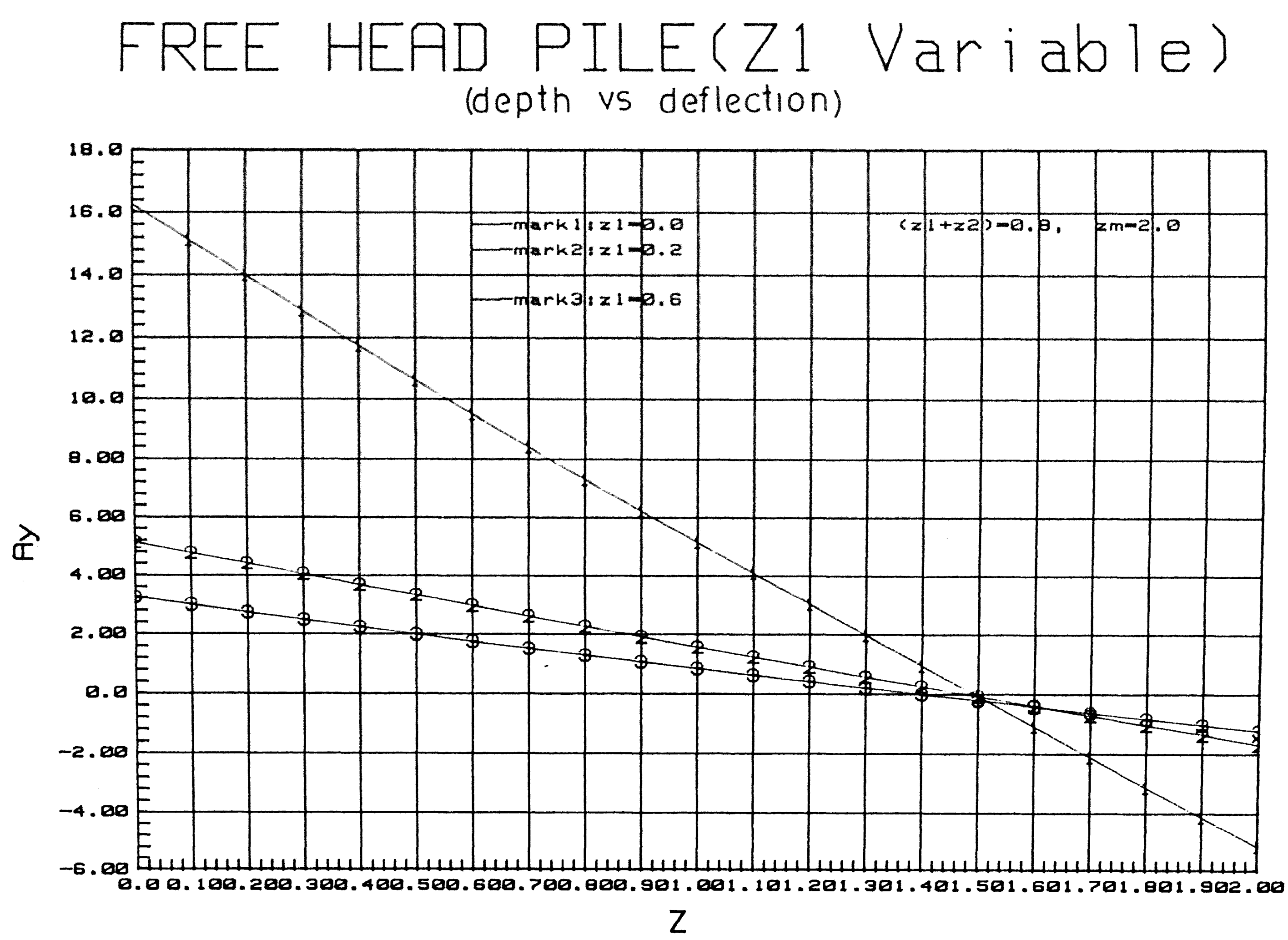
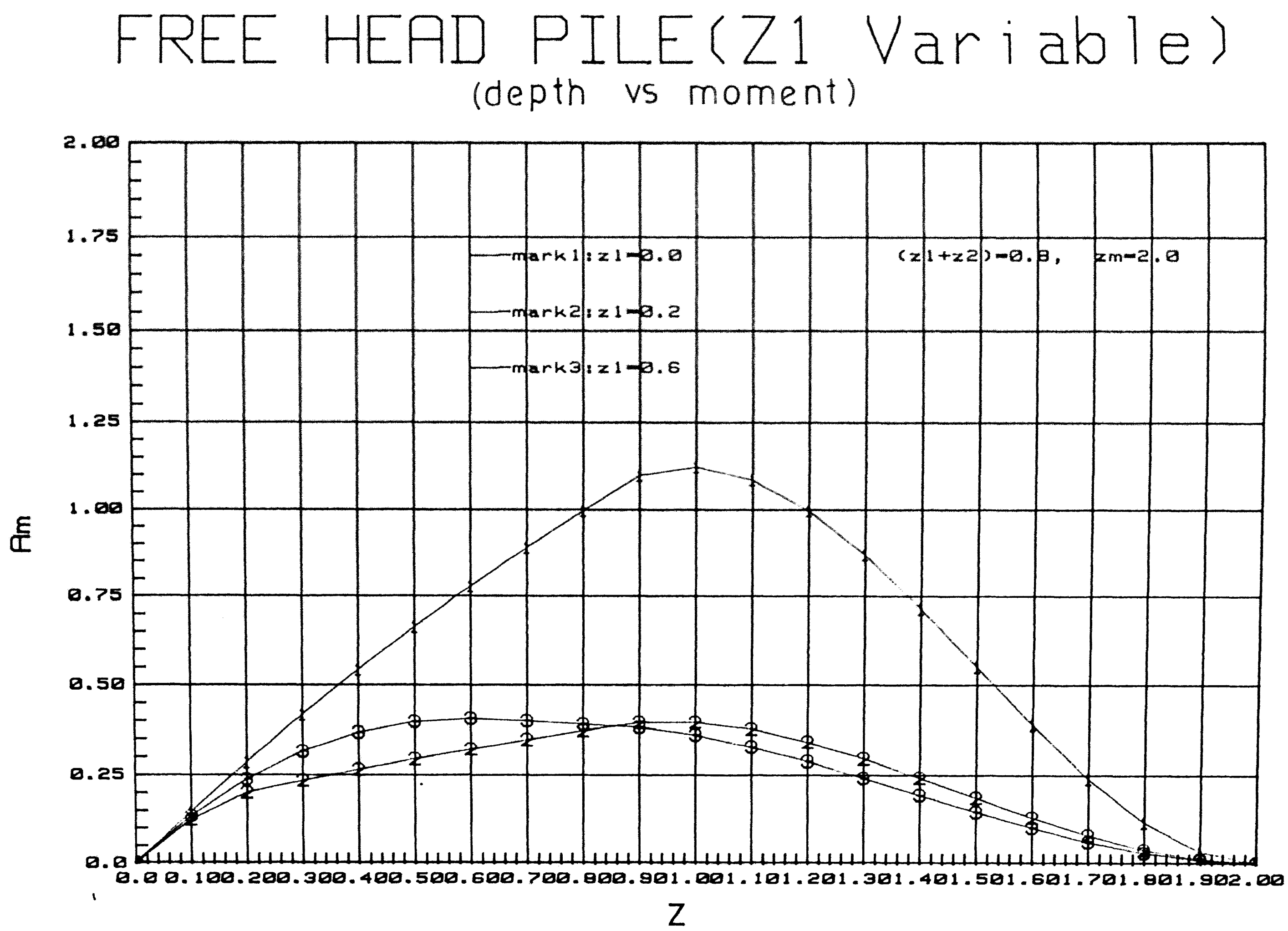

FREE HEAD PILE(Z1 Variable)
(depth vs deflection)
mark1:z1=0.0
mark2:z1=0.2
mark3:z1=0.6
(z1+z2)=0.8, zm=2.0
Ay
Z
18.0
16.0
14.0
12.0
10.0
8.00
6.00
4.00
2.00
0.0
-2.00
-4.00
-6.00
0.0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00 1.10 1.20 1.30 1.40 1.50 1.60 1.70 1.80 1.90 2.00

FIG 4·12



FREE HEAD PILE(Z1 Variable)
(depth vs moment)
mark1:z1=0.0
mark2:z1=0.2
mark3:z1=0.6
(z1+z2)=0.8, zm=2.0
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.0
Am
0.0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00 1.10 1.20 1.30 1.40 1.50 1.60 1.70 1.80 1.90 2.00
Z

FIG 4.13

FREE HEAD PILE

(depth vs deflection, moment)

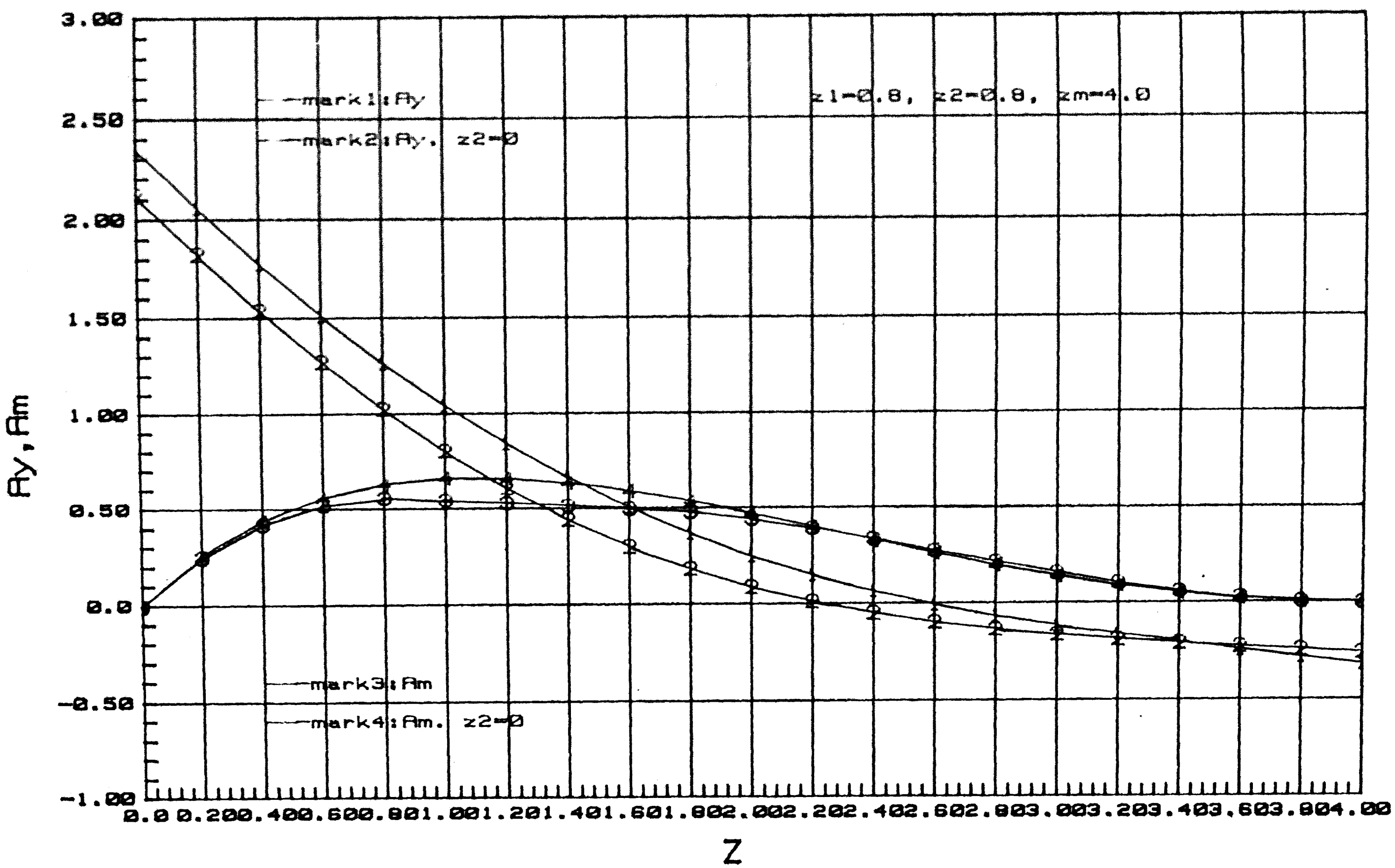


FIG 4.14

FREE HEAD PILE(Z1 Variable)

(depth vs deflection)

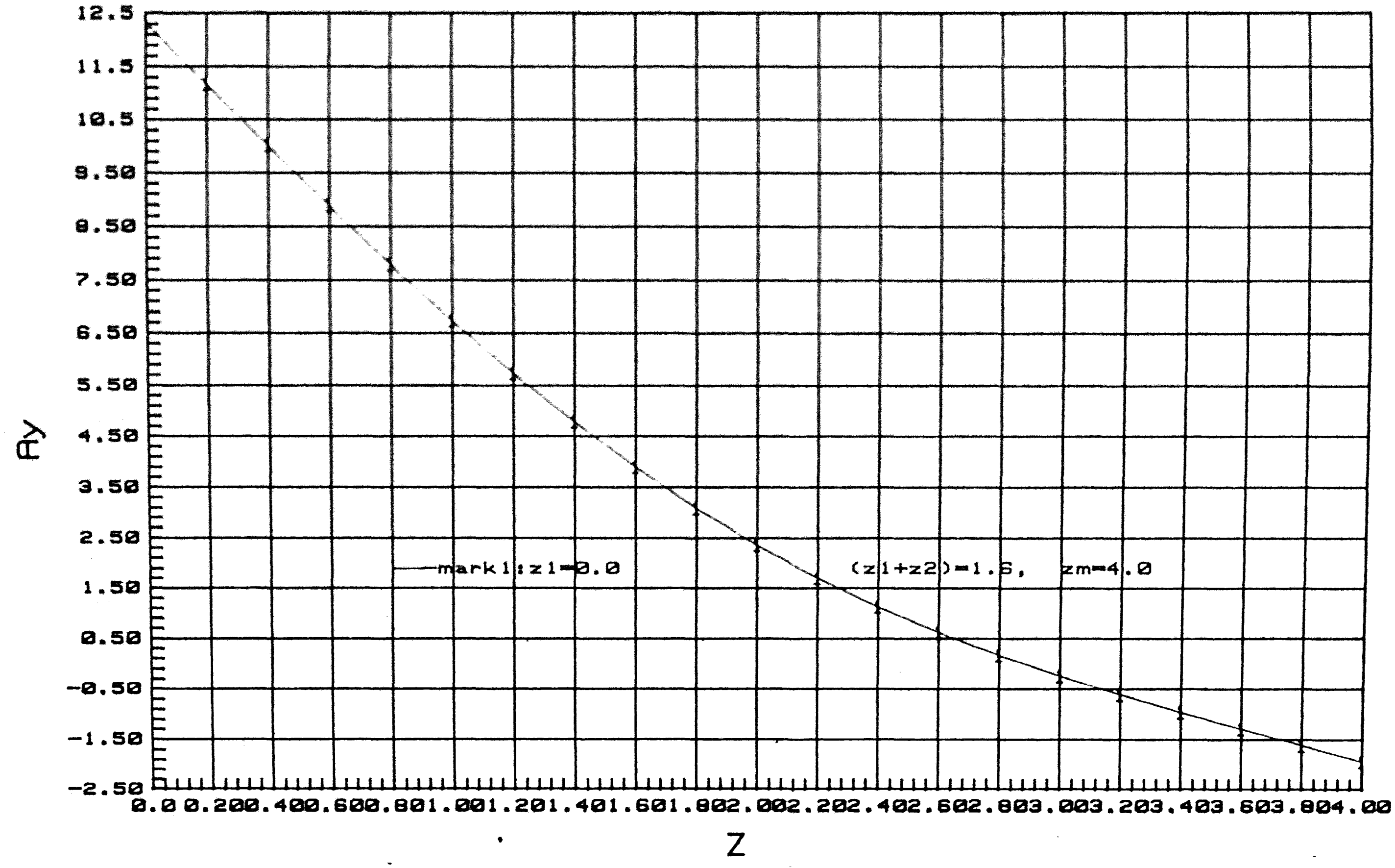
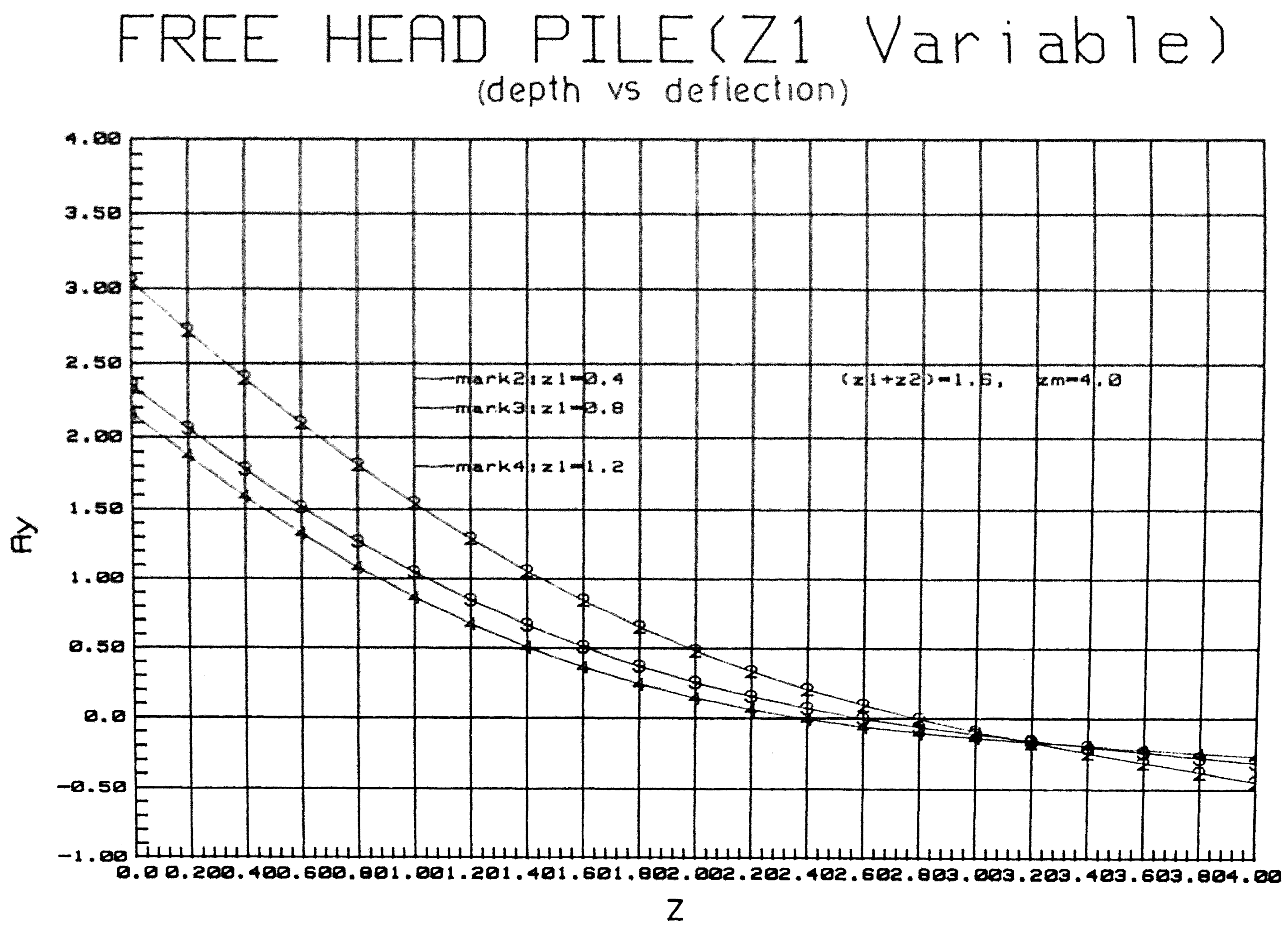


FIG 4·15(a)



FREE HEAD PILE(Z1 Variable)
(depth vs deflection)
Ay
4.00
3.50
3.00
2.50
2.00
1.50
1.00
0.50
0.0
-0.50
-1.00
0.0 0.20 0.40 0.60 0.80 1.00 1.20 1.40 1.60 1.80 2.00 2.20 2.40 2.60 2.80 3.00 3.20 3.40 3.60 3.80 4.00
Z
mark2:z1=0.4
mark3:z1=0.8
mark4:z1=1.2
(z1+z2)=1.6, zm=4.0

FIG 4·15(b)

FREE HEAD PILE(Z1 Variable)

(depth vs moment)

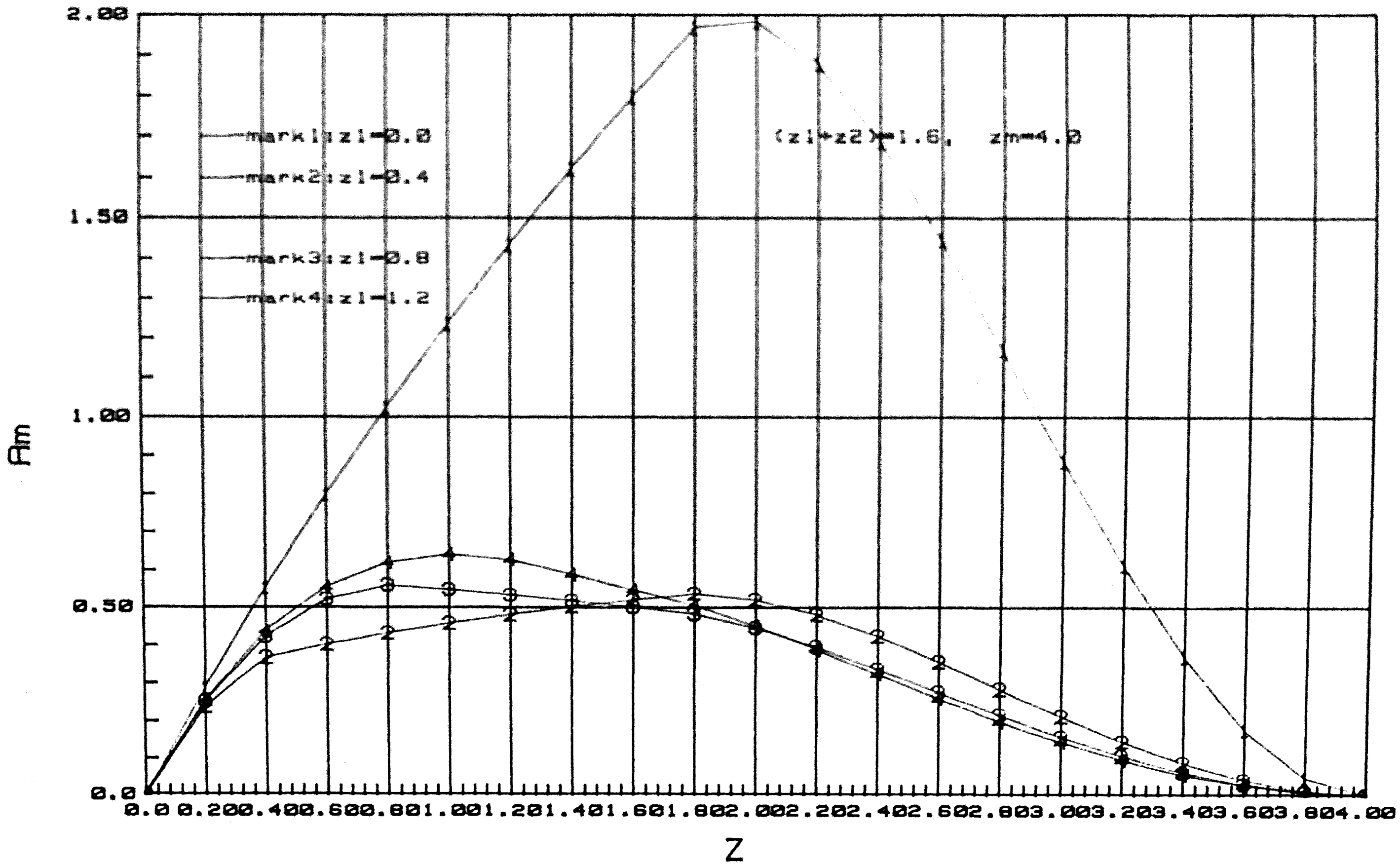


FIG 4·16

FREE HEAD PILE(Z1 Constant)

(depth vs deflection)

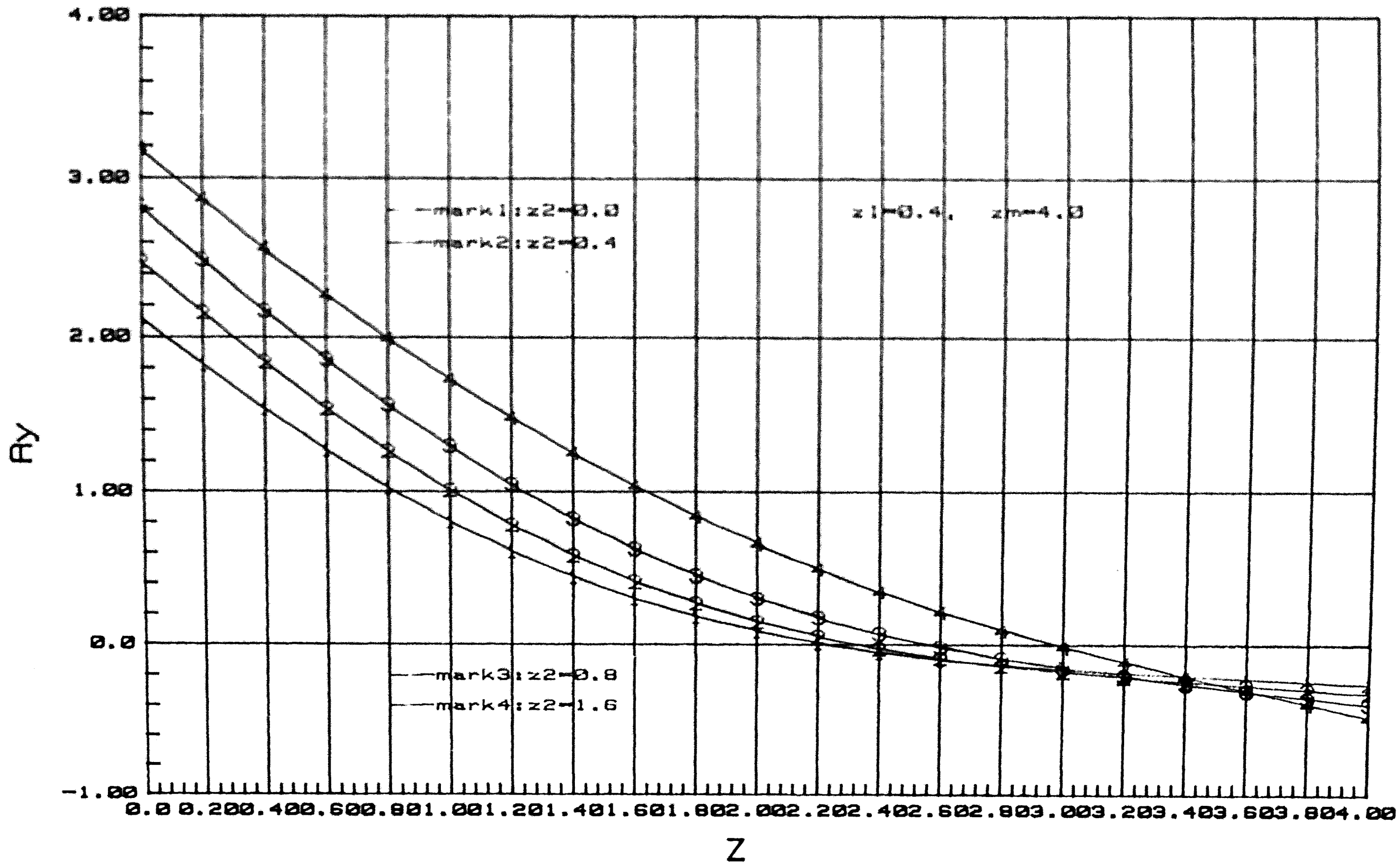


FIG 4.17

FREE HEAD PILE(Z1 Constant)
(depth vs moment)

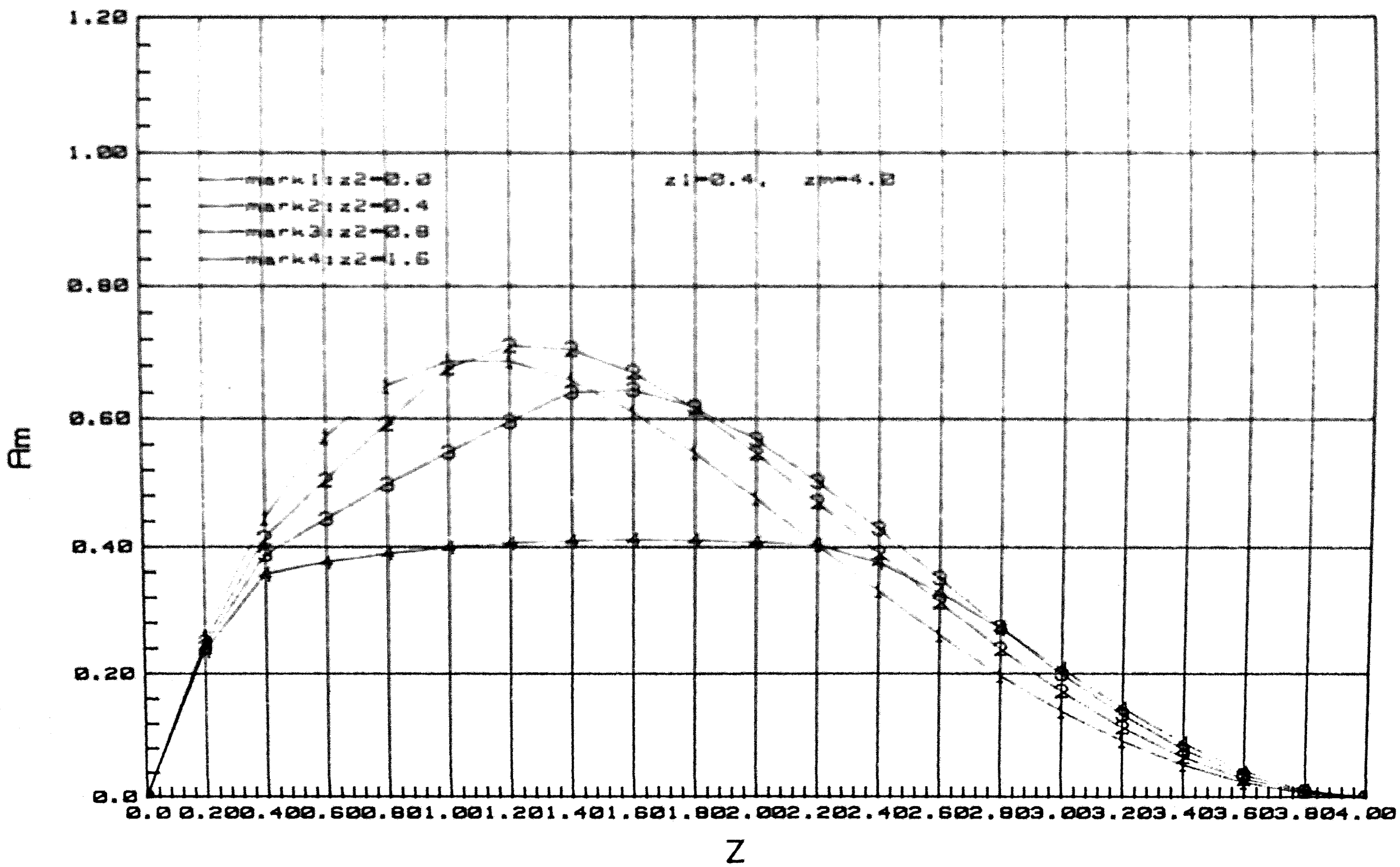


FIG 4·18

FIXED HEAD PILE

(depth vs deflection, moment)

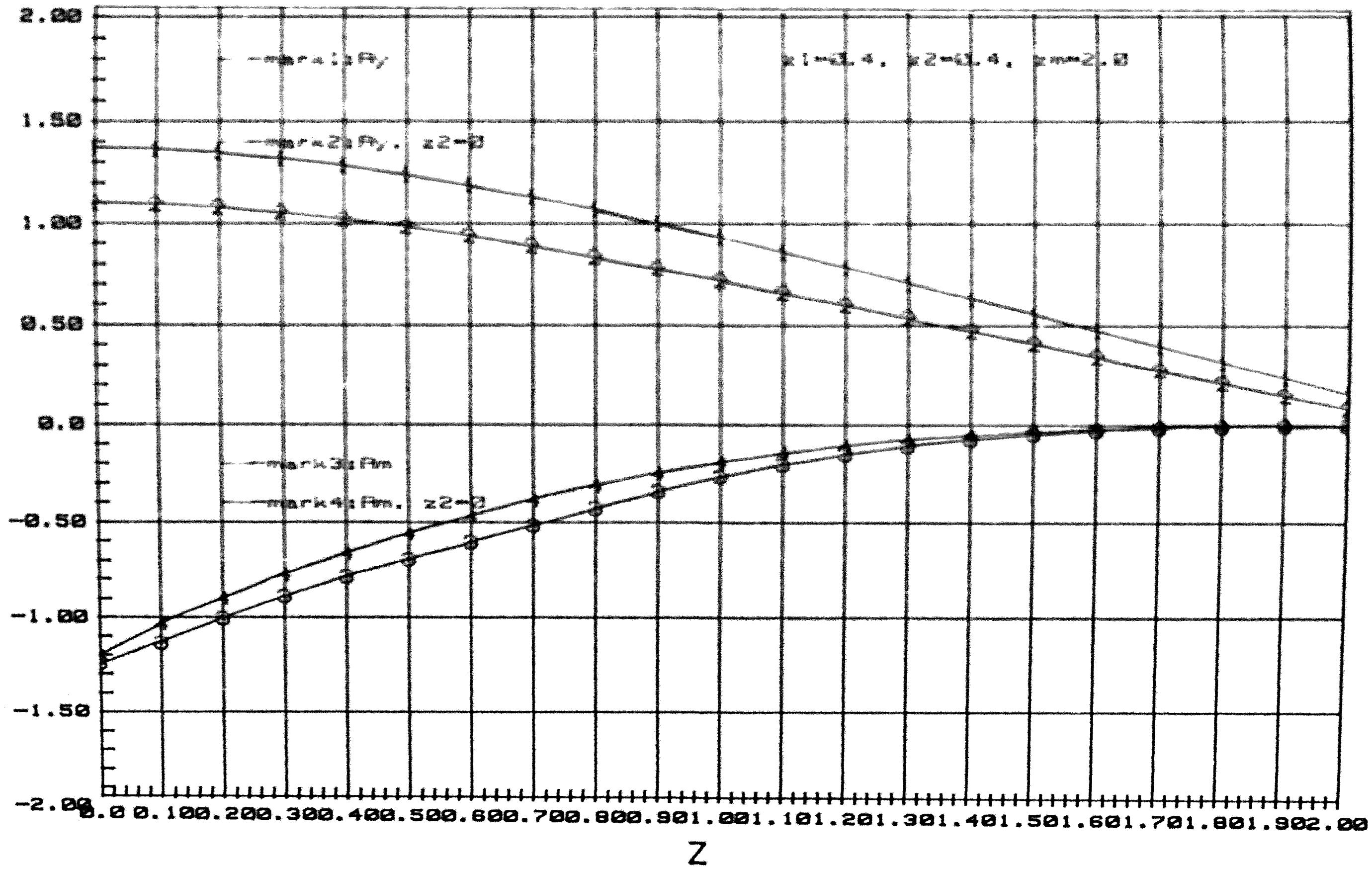


FIG 4·19

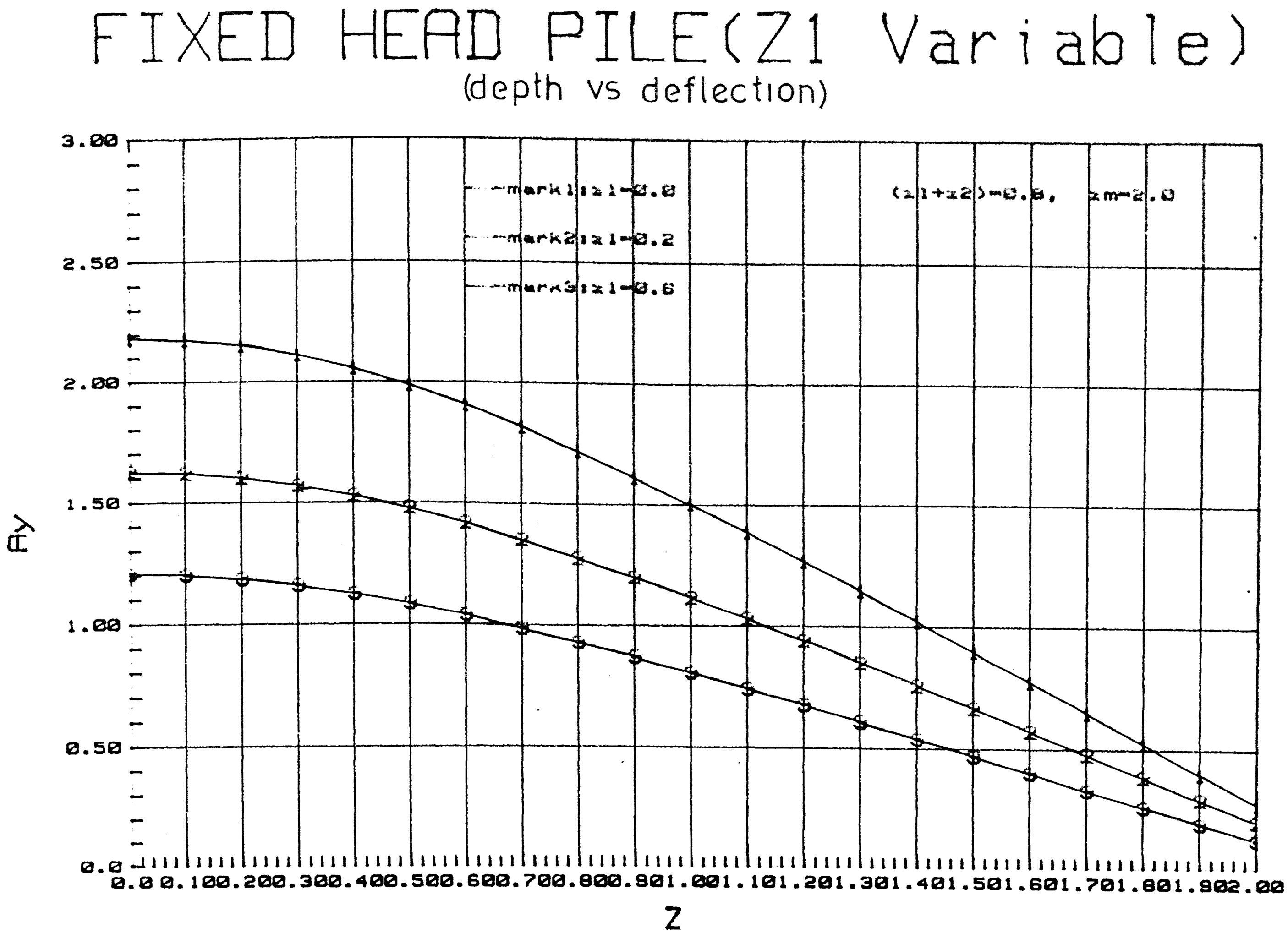
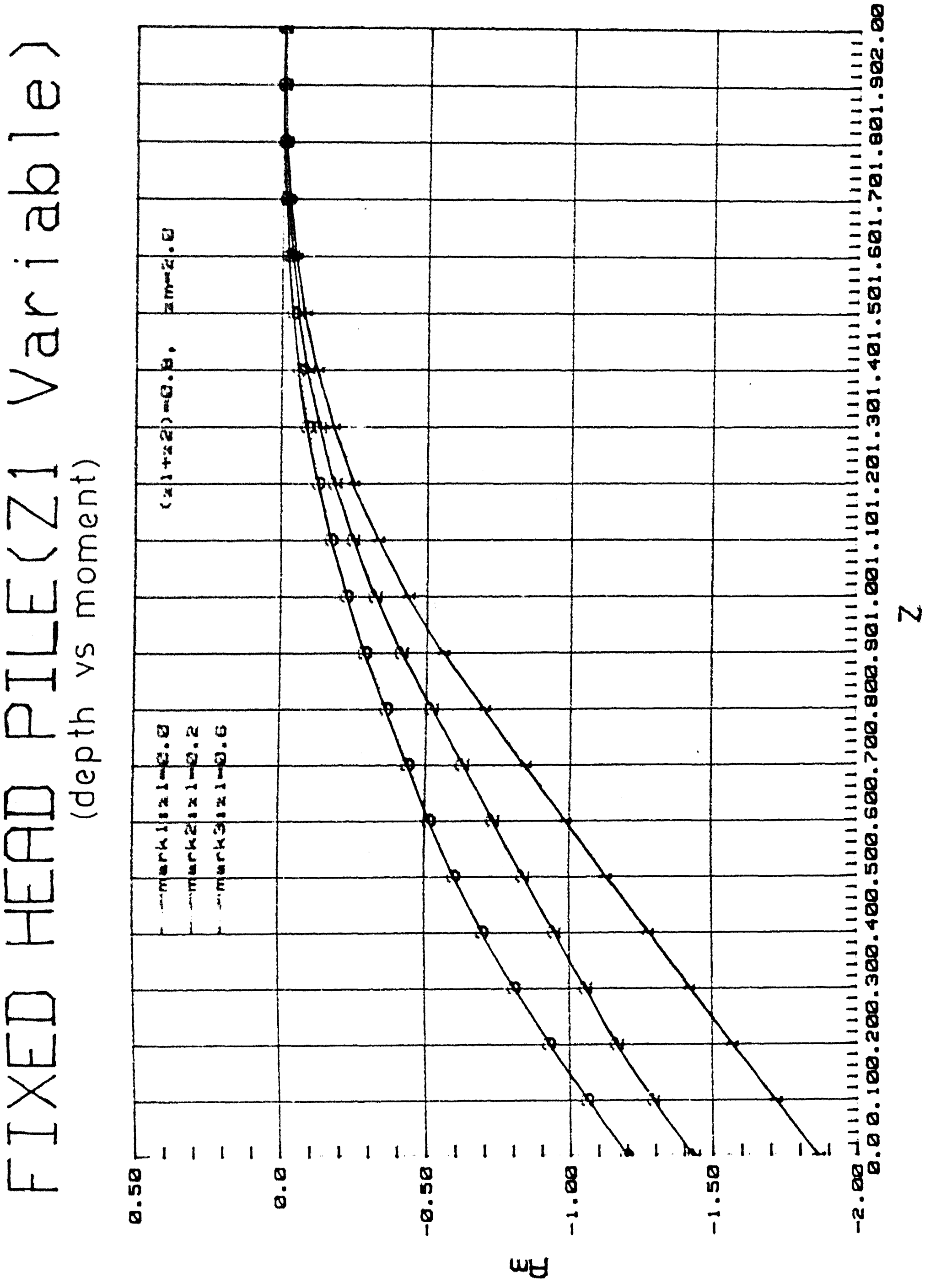

FIXED HEAD PILE(Z1 Variable)
(depth vs deflection)
mark1:z1=0.0
mark2:z1=0.2
mark3:z1=0.6
(z1+z2)=0.8, zm=2.0
3.00
2.50
2.00
1.50
1.00
0.50
0.0
Ay
0.0 0.10 0.20 0.30 0.40 0.50 0.60 0.70 0.80 0.90 1.00 1.10 1.20 1.30 1.40 1.50 1.60 1.70 1.80 1.90 2.00
Z

FIG 4·20



FIXED HEAD PILE(Z1 Variable)
(depth vs moment)
mark1:z1=0.0
mark2:z1=0.2
mark3:z1=0.6
(z1+z2)=0.8, zm=2.0
0.50
0.0
-0.50
-1.00
-1.50
-2.00
Am
0.00.100.200.300.400.500.600.700.800.901.001.101.201.301.401.501.601.701.801.902.00
Z

FIG 4.21

FIXED HEAD PILE

(depth vs deflection, moment)

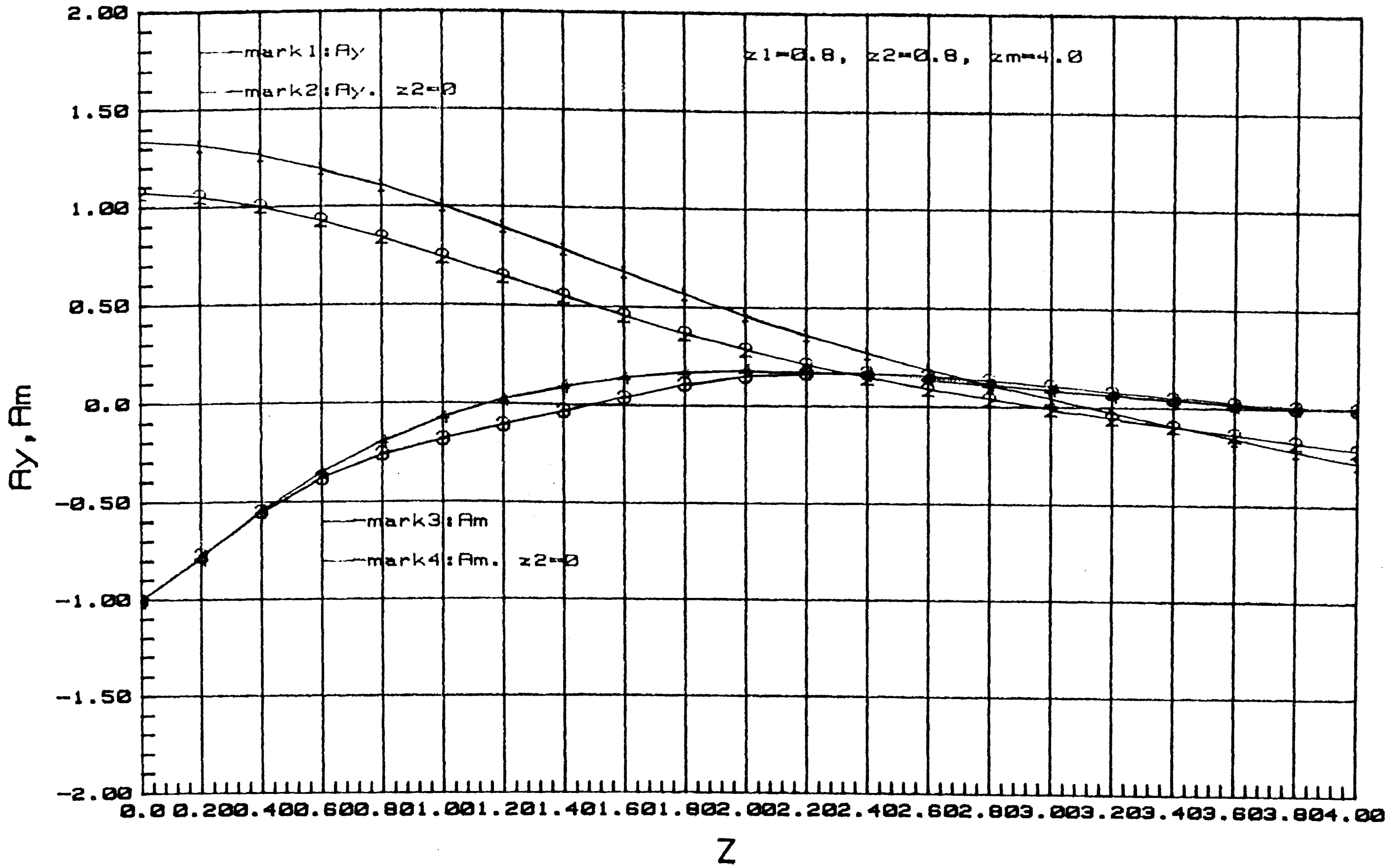


FIG 4·22

FIXED HEAD PILE(Z1 Variable)

(depth vs deflection)

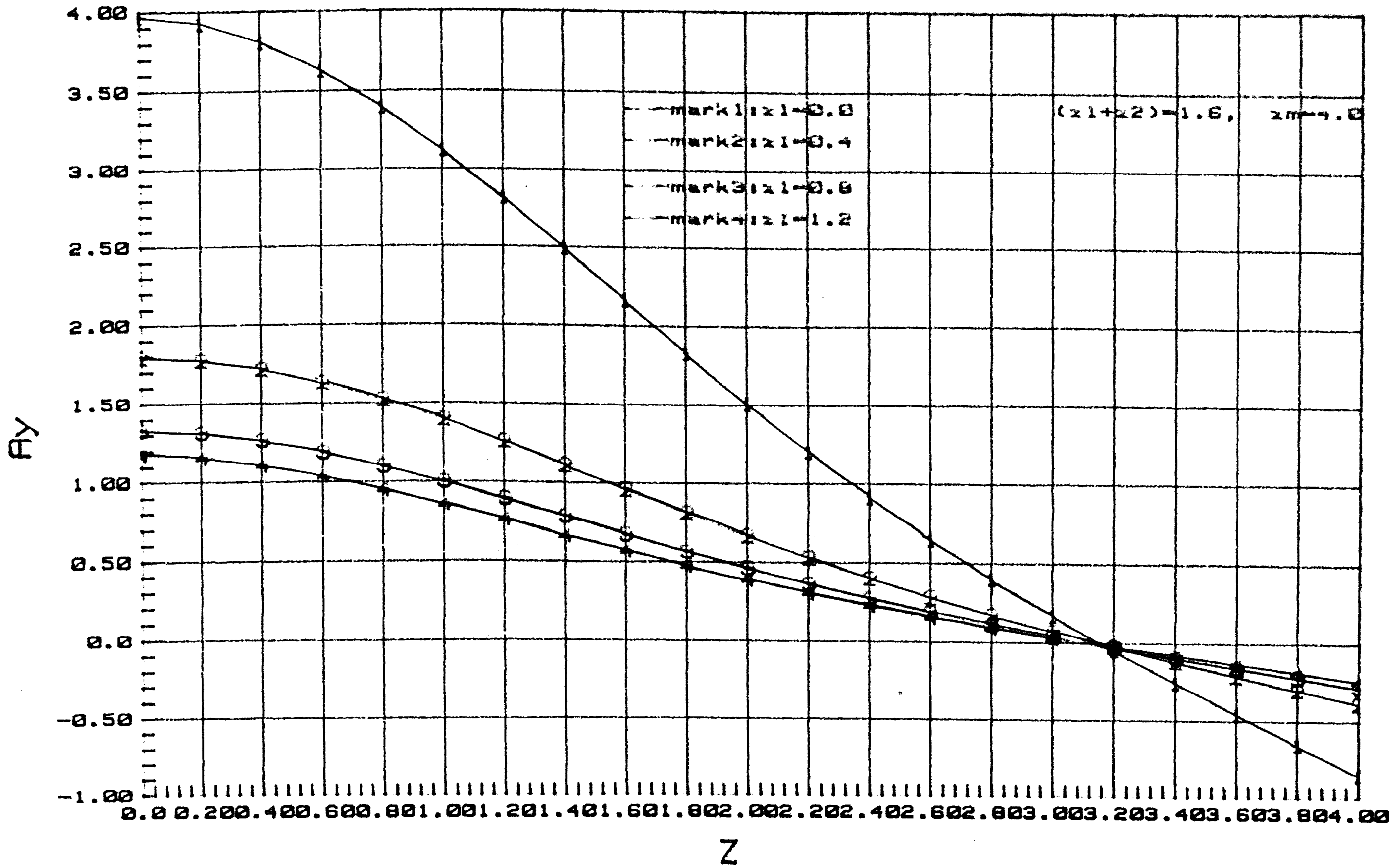


FIG 4.23

FIXED HEAD PILE(Z1 Constant)

(depth vs deflection)

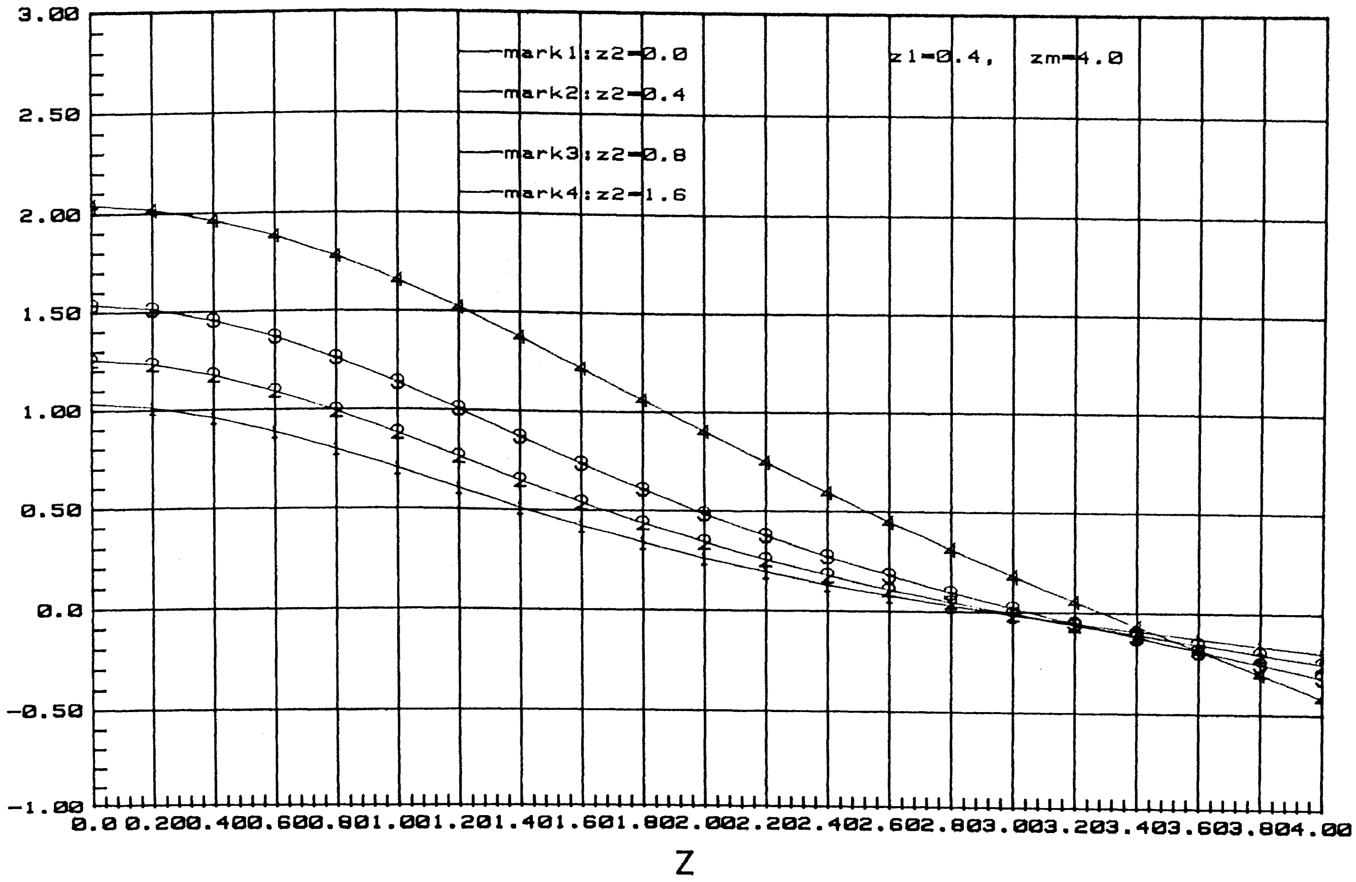


FIG 4.25

FIXED HEAD PILE(Z1 Constant)
(depth vs moment)

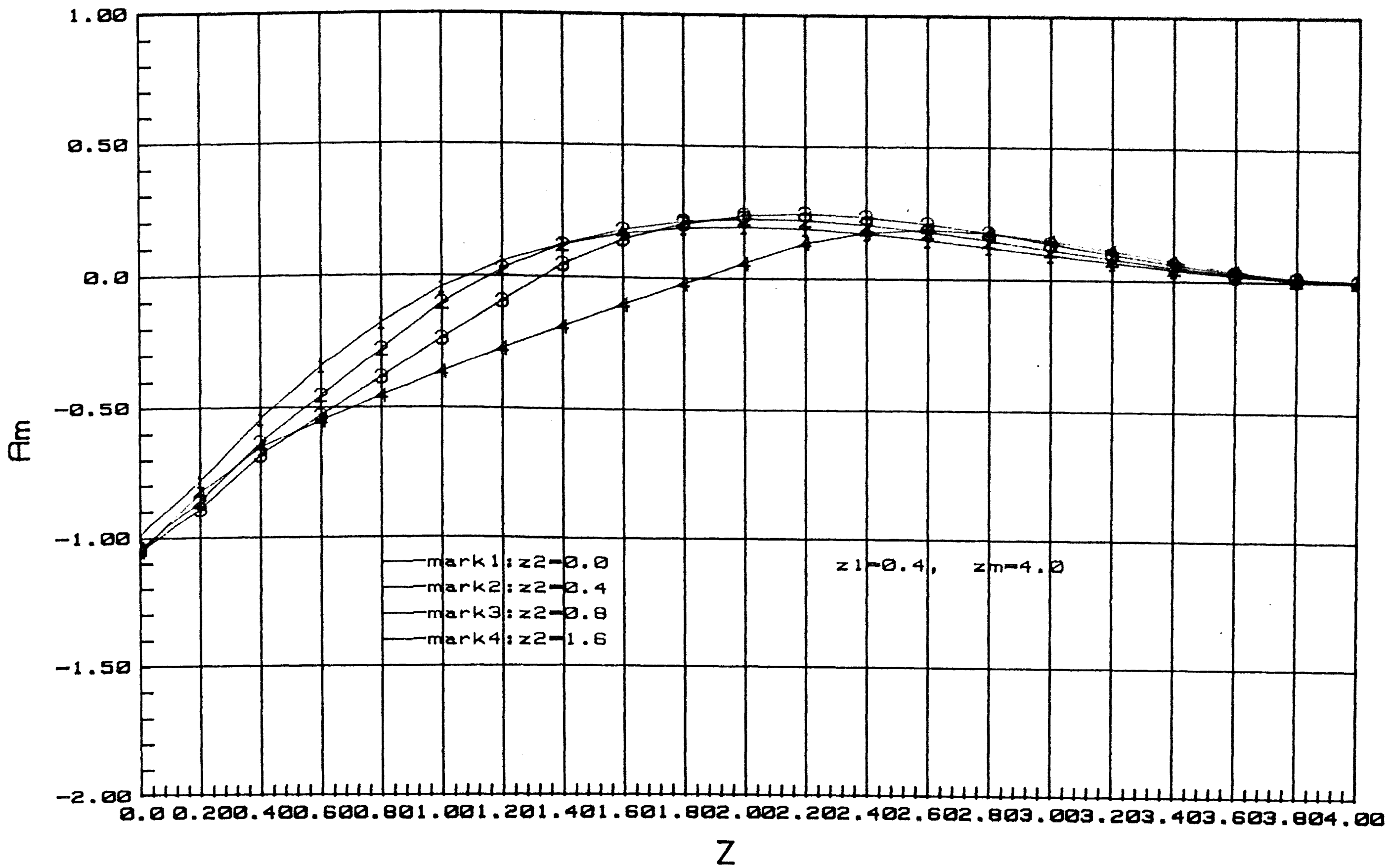


FIG 4.26

Table 4.1 : Load Capacity Variation for Free Head Short Pile

(R = NLLC for given values of NCD and NLD / NLLC with NLD = 0)

	R						
NLD	0.0	0.2	0.4	0.6	0.8	0.9	1.0
(NCD)							
0.0	1	0.90	0.65	0.36	0.10	0.04	0
0.1	1	0.82	0.52	0.25	0.06	0.04	-
0.2	1	0.76	0.46	0.20	0.12	--	-
0.4	1	0.70	0.46	0.38	--	--	-

Table 4.2 : Load Capacity Variation for Fixed Head Short Pile

	R						
NLD	0.0	0.2	0.4	0.6	0.8	0.9	1.0
(NCD)							
0.0	1	0.96	0.83	0.63	0.36	0.20	0
0.1	1	0.92	0.75	0.53	0.20	0.03	-
0.2	1	0.90	0.68	0.40	0.053	--	-
0.4	1	0.80	0.53	0.16	--	--	-

Table 4.3 : Variation of Non-dimensional lateral load capacity for Free Head Long Pile

NLLC

NYC	OHR=0.0 .013	.42	(%)	OHR=0.1 .013	.42	%	OHR=0.2 .013	.42	(%)
(NLD)									
0.0	.075	.85	(1033)	.05	.77	(1440)	.04	.68	(1600)
0.2	.05	.80	(1500)	.03	.70	(2233)	.02	.65	(3150)
0.4	.03	.70	(2233)	.02	.60	(2930)	.016	.55	(3340)
0.6	.02	.57	(2750)	.015	.53	(3400)	.015	.48	(3100)
%	-73	-33		-70	-32		-62.5	-30	

Table 4.4 : Variaton of Non-dimensional lateral load capacity for Fixed Head Long Pile

NLLC

NYC	OHR=0.0 .013	.42	(%)	OHR=0.1 .013	.42	%	OHR=0.2 .013	.42	(%)
(NLD)									
0.0	.15	1.37	(813)	.10	1.25	(1150)	.07	1.12	(1500)
0.2	.10	1.30	(1200)	.07	1.20	(1614)	.05	1.10	(2100)
0.4	.06	1.2	(1900)	.05	1.10	(2100)	.04	1.00	(2400)
0.6	.05	1.05	(2000)	.04	0.95	(2200)	.03	0.90	(2900)
%	-66	-23		-60	-24		-57	-20	

Table 4.5 : Effect of Soil Cover on the Flexural Behaviour of Free Head Short Pile

$(Z_1+Z_2)=0.8$, Zm=2.0

Z_1	Ratio $(Ay)_{top}$	Ratio $(Am)_{max}$
0.8	1.00	1.00
0.6	1.06	0.93
0.2	1.73	0.88
0.0	5.40	2.60

Table 4.6 : Effect of Soil Cover on the Flexural Behaviour of Free Head Long Pile

$(Z_1+Z_2) = 1.6$, $Z_m = 4.0$

Z_1	Ratio $(Ay)_{top}$	Ratio $(Am)_{max}$
1.6	1.00	1.00
1.2	1.03	0.98
0.8	1.12	0.85
0.4	1.45	0.83
0.0	5.95	3.00

Table 4.7 : Effect of Liquefied Depth of Soil on the Flexural Behaviour of Free Head Long Pile

$Z_1 = 0.4$, $Z_m = 4.0$

Z_2	Ratio $(Ay)_{top}$	Ratio $(Am)_{max}$
0.0	1.00	1.00
0.4	1.18	1.04
0.8	1.33	0.97
1.6	1.52	0.62

Table 4.8 : Effect of Soil Cover on the Flexural Behaviour of Fixed Head Short Pile

$(Z_1+Z_2) = 0.8$, $Z_m = 2.0$

Z_1	Ratio $(Ay)_{top}$	Ratio $(Am)_{max}$
0.8	1.00	1.00
0.6	1.06	0.96
0.2	1.44	1.15
0.0	1.95	1.48

Table 4.9 : Effect of Soil Cover on the Flexural Behaviour of Fixed Head Long Pile

$(Z_1+Z_2) = 1.6$, $Z_m = 4.0$

Z_1	Ratio $(Ay)_{top}$	Ratio $(Am)_{max}$
1.6	1.00	1.00
1.2	1.12	0.98
0.8	1.25	0.98
0.4	1.68	1.05
0.0	3.70	2.20

Table 4.10 : Effect of Liquefied Soil Depth on the Flexur Behaviour of Fixed Head Long Pile

$Z_1 = 0.4$, $Z_m = 4.0$

Z_2	Ratio $(Ay)_{top}$	Ratio $(Am)_{max}$
0.0	1.00	1.00
0.4	1.22	1.07
0.8	1.50	~1.07
1.6	2.00	~1.07

5. CONCLUSION

Based on the results and discussions presented in the Section 4 the following generalized conclusion can be drawn.

(1) The lateral load capacity of piles decreases substantially with the increase in the liquefied soil depth. The decrease is more prominent when the liquefaction starts right from the ground surface.

(2) Pile head deflections increase as the soil along the embedded length of the pile liquefies and is a function of various factors like the liquefied depth of the soil, length of the pile, soil cover, applied loads etc.

(3) The beneficial effect of the ground improvement near the ground surface is significant.

(4) The presented design charts and the computer program can be used fruitfully in determining the pile load capacity for short and long piles and finding the flexural response of a pile under various conditions of soil liquefaction and the pile head fixity.

(5) The presented design charts and the computer program are helpful in the quantitative analysis of the various factors affecting the pile capacity and the flexural behaviour under the various soil and pile conditions.

(6) Providing fixed head pile is more beneficial under the conditions of soil liquefaction than free head pile. This observation is similar to the one made for the non-liquefied case.

6. SCOPE OF FUTURE STUDY

(1) Analytical study for the determination of the lateral load capacity and flexural behaviour of a pile group under the various conditions of soil liquefaction and pile head fixity.

(2) Experimental studies can be conducted with model piles to find the effect of soil liquefaction and soil movement on the pile behaviour.

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```
c       This program finds the lateral load capacity of a pile when the
c       liquefaction has occured upto a certain depth of the pile. The
c       pile can be either a free head pile or a fixed head pile.While
c       executing the program the relevent data is asked for.
c       The variables asked for are pile diameter,depth where liquefac-
c       tion starts i.e. l1, depth of liquefaction l2, total length of
c       pile l,eccentricity e, unit weight of soil, passive earth pre-
c       ssure cofficient kp,yield moment of the pile my .
c       Program starts from here.
        real d,l1,l2,l3,l4,l,e,g,kp,den,vo,cd,my,mmax
        read*, d,l1,l2,l3,l,e,g,kp,den,vo,cd,my
        a = 1.5*g*d*kp*l1*l1
        b = 3*g*d*(l1+l2)*l3*kp
        c = 1.5*g*d*kp*l3*l3
        fd = den*vo*vo*d*l2*cd/6.0
        fd = fd/1000
        write(*,*)'if free head pile then press 1 else 2'
        read*,itype
c       program starts for free head piles
c       assuming it to be ashort pile
        if (itype.eq.1) then
        hu= (a*(l-0.67*l1)+b*0.5*l3+c*0.33*l3-fd*(0.75*l2+l3))/(e+l)
c        to find maximum moment equate SF to zero
c       assuming maximum moment to be in the l1 part
          f = 0.82*sqrt(hu/(d*kp*g))
          if(f.le.l1) then
             mmax = hu*(e+0.67*f)
                if(mmax.le.my) then
                    hu = hu
                    write(*,*)' the pile is behaving as a short pile'
                    write(*,*)'the maximum moment reached in kN is'
                    write(*,*) mmax
                    else
                    y1 = my/(1.5*g*d*kp)
                    f=0
50                  f=f+0.1
                    f1=f*f*(e+0.67*f)
                      if(f1.lt.y1) then
                        go to 50
                        else
                        write(*,*) 'pile behaving as a long pile'
                        write(*,*)' and the pile has yielded'
                        hu=1.5*g*d*kp*f*f
                      endif
                endif
          endif
c               maximum moment in l3 part
         if(f.gt.l1) then
             c1=(a-hu-fd)*l3*l3
             x= (sqrt((b*l3)**2-4*c*c1)-b*l3)/(2*c)
             mmax =hu*(e+l1+l2+x)+fd*(0.75*l2+x)-a*(x+l2+0.33*l1)
     *       -0.5*b*x**2/l3 -(0.33*c*x**3)/(l3*l3)
                if(mmax.le.my) then
                   hu=hu
                   mmax=mmax
                   write(*,*) 'pile behaving as a short pile'
                   write(*,*) 'the maximum moment reached in kN-m is '
                   write(*,*) mmax
                   else
```

```
c                       yielding case,behaving as a long pile
                      x=0
 60                   x=x+0.01
                      hu=(my+a*(0.33*l1+l2+x)+0.5*b*x**2/l3
     *                +0.33*c*x**3/(l3*l3))/(e+l1+l2+x)
                       c1 =(a-hu-fd)*l3*l3
                      x1=(sqrt((b*l3)**2-4*c*c1)-b*l3)/(2*c)
                         if(x.gt.x1) then
                         hu=hu
                         write(*,*)'the pile has yielded .The maximum'
                         write(*,*)'moment has reached yield capacity'
                         write(*,*)'the depth of yield  is x =',x
                         else
                         go to 60
                         endif
                    endif
                 endif
                 write(*,*) 'the ultimate lateral capacity in kN is'
                 write(*,*) hu
                 endif
c             **********************************************************
                    if(itype.eq.2) then
c             here program starts for fixend end pile
c             ---------------------------------------
c            assuming short pile
             hu1=a+b+c-fd
             mmax=a*(e+0.67*l1)+b*(l-0.5*l3)+c*(l-0.33*l3)
     *       -fd*(e+l1+0.25*l2)
             if(mmax.le.my) then
             hu=hu1
             mmax=mmax
             write(*,*)'the fixed  head pile behaving as short pile'
             write(*,*)'the maximum moment reached in kN-m is'
             write(*,*) mmax
             write(*,*)'the lateral load capacity in kN is'
             write(*,*) hu
             endif
               if(mmax.gt.my) then
               hu=hu1-(my+mmax)/(e+l)
c            to check that maximum moment at some depth is in limit'
              f = 0.82*sqrt(hu/(d*kp*g))
              if(f.le.l1) then
                 mmax = hu*(e+0.67*f)
                    if(mmax.le.my) then
                       hu = hu
                       write(*,*)' the pile is behaving as intermediate pil
                       write(*,*)'the maximum moment reached in kN is'
                       write(*,*) mmax
                       else
                       y1 =2*my/(1.5*g*d*kp)
                       f=0
 70                    f=f+0.1
                       f1=f*f*(e+0.67*f)
                         if(f1.lt.y1) then
                           go to 70
                           else
                           write(*,*) 'pile behaving as a long pile'
                           write(*,*)'the pile has yielded at two points'
                           hu=1.5*g*d*kp*f*f
                         endif
                    endif
               endif
c                   maximum moment in l3 part
             if(f.gt.l1) then
                 c1=(a-hu-fd)*l3*l3
                 x= (sqrt((b*l3)**2-4*c*c1)-b*l3)/(2*c)
```

```
mmax =hu*(e+l1+l2+x)+fd*(0.75*l2+x)-a*(x+l2+0.33*l1)
-0.5*b*x**2/l3 -(0.33*c*x**3)/(l3*l3)
   if(mmax.le.my) then
       hu=hu
       mmax=mmax
       write(*,*) 'pile behaving as a short pile'
       write(*,*) 'the maximum moment reached in kN-m is
       write(*,*) mmax
       else
            yielding case,behaving as a long pile
       x=0
       x=x+0.01
       hu=(2*my+a*(0.33*l1+l2+x)+0.5*b*x**2/l3
       +0.33*c*x**3/(l3*l3))/(e+l1+l2+x)
        c1 =(a-hu-fd)*l3*l3
       x1=(sqrt((b*l3)**2-4*c*c1)-b*l3)/(2*c)
           if(x.gt.x1) then
           hu=hu
           write(*,*)'the pile has yielded at two points the
           write(*,*)'maximum moment has reached yield value
           else
           go to 80
           endif
   endif
 endif
 write(*,*) 'the ultimate lateral capacity in kN is'
 write(*,*) hu
 endif
   endif
 stop
 end
```

An example has been solved to show the use of the the program developed for the ultimate lateral load capacity for the liquefied case.The input data given is the following .The symbols used are the same as that given in the program
d = 0.5m, l1 = 3m, l2 = 4m, l3 = 5m, l = 12m, e = 0.5m, g = 8kN/cu.m, Kp = 3, den = 1800kg/cu.m, Vo = 0.5m/s, Cd = 0.5 my = 500kNm.
For this input for free head pile case the output comes as following.
'The pile has yielded. The maximum moment has reached yield capacity.The ultimate lateral capacity reached in kN is 174.41.'

```
c
c          of deflection and bending moment for a laterally loaded pile
c          for a FREE HEAD head pile case.
c          The variables to be given are r1,which is equal to 1, z1-
c          which is the non-dimensional depth from the ground surface
c          to where liquefaction starts,z2- which is the depth of liqu-
c          efaction zm- is the total length  of pile np- number of sub-
c          divisions.
           parameter(nmax=200)
           double precision a(200,200),p(200,1),x(200)
           double precision d(200),am(200),av(200)
           read*,r1,z1,z2,zm,np
                   do 10 i=1,np+1
                   do 5 j=1,np+1
                   a(i,j)=0
 5                 continue
 10                continue
                do  20 i=1,np+1
                 p(i,1)=0
 20             continue
          np1=ifix(z1*np/zm+1)
          np2=ifix((z1+z2)*np/zm+1)
          del=zm/np
 c                 program
          r2=r1
          r3=1.25*r1
            p(1,1)=del**3
          a(1,1)=(2+r1*del**4)
          a(1,2)=-4
          a(1,3)=2
 c        ******
          a(2,1)=-2
          a(2,2)=(5+r1*del**4)
          a(2,3)=-4
          a(2,4)=1
             do 30 i=3,np1
             a(i,i-2)=1
             a(i,i-1)=-4
             a(i,i)=(6+r1*del**4)
             a(i,i+1)=-4
             a(i,i+2)=1
 30          continue
             do 40 i=np1+1,np2
             a(i,i-2)=1
             a(i,i-1)=-4
             a(i,i)=(6+r2*del**4)
             a(i,i+1)=-4
             a(i,i+2)=1
 40          continue
             do 50 i=np2+1,np-1
             a(i,i-2)=1
             a(i,i-1)=-4
             a(i,i)=(6+r3*del**4)
             a(i,i+1)=-4
             a(i,i+2)=1
 50          continue
 c         **********
         a(np,np-2)=1
         a(np,np-1)=-4
         a(np,np)=(5+r3*del**4)
         a(np,np+1)=-2
          *********
```

```
*       am(1)=0
        do 51 i= 2,np
        am(i)=(x(i+1)-2*x(i)+x(i-1))/(del**2)
51      continue
        am(np+1)=0
        av(1)=1
        av(2)=1
        do 52 i=3,np-1
        av(i)=(-x(i-2)+2*x(i-1)-2*x(i+1)+x(i+2))/(del**3)
52      continue
        av(np)=0
        av(np+1)=0
        do 53 i=1,np+1
        d(i)=(i-1)*del
53      continue
        do 60 i=1,np+1
        write(*,61) d(i),am(i)
60      continue
61      format(2x,2f9.3)
        stop
        end
        subroutine gauss(a,p,x,n)
        double precision a(200,200),p(200,1),x(200)
        do 200 i=1,n
           do 200 j=1,n
              if (i.ne.j) then
              p(j,1)=p(j,1)-a(j,i)*p(i,1)/a(i,i)
              do 100 k=n,1,-1
100              a(j,k)=a(j,k)-a(i,k)*a(j,i)/a(i,i)
              endif
200           continue
        do 300 i=1,n
300     x(i)=  p(i,1)/a(i,i)
        return
        end
```

```
c          of deflection and bending moment for a laterally loaded pile
c          for a FIXED HEAD head pile case.
c          The variables to be given are r1,which is equal to 1, z1-
c          which is the non-dimensional depth from the ground surface
c          to where liquefaction starts,z2- which is the depth of liqu-
c          efaction zm- is the total length  of pile np- number of sub-
c          divisions.
           parameter(nmax=200)
           double precision a(200,200),p(200,1),x(200)
           double precision d(200),am(200),av(200)
           read*,r1,z1,z2,zm,np
                   do 10 i=1,np+1
                   do 5 j=1,np+1
                   a(i,j)=0
 5                 continue
 10                continue
                do  20 i=1,np+1
                 p(i,1)=0
 20             continue
          np1=ifix(z1*np/zm+1)
          np2=ifix((z1+z2)*np/zm+1)
          del=zm/np
 c                 program
          r2=r1/20
          r3=1.25*r1
            p(1,1)=del**3
          a(1,1)=(6+r1*del**4)
          a(1,2)=-8
          a(1,3)=2
 c        ******
          a(2,1)=-4
          a(2,2)=(7+r1*del**4)
          a(2,3)=-4
          a(2,4)=1
             do 30 i=3,np1
             a(i,i-2)=1
             a(i,i-1)=-4
             a(i,i)=(6+r1*del**4)
             a(i,i+1)=-4
             a(i,i+2)=1
 30          continue
             do 40 i=np1+1,np2
             a(i,i-2)=1
             a(i,i-1)=-4
             a(i,i)=(6+r2*del**4)
             a(i,i+1)=-4
             a(i,i+2)=1
 40          continue
             do 50 i=np2+1,np-1
             a(i,i-2)=1
             a(i,i-1)=-4
             a(i,i)=(6+r3*del**4)
             a(i,i+1)=-4
             a(i,i+2)=1
 50          continue
 c         **********
         a(np,np-2)=1
         a(np,np-1)=-4
         a(np,np)=(5+r3*del**4)
         a(np,np+1)=-2
          *********
```

```
      am(1)=0
      do 51 i= 2,np
      am(i)=(x(i+1)-2*x(i)+x(i-1))/(del**2)
51    continue
      am(np+1)=0
      av(1)=1
      av(2)=1
      do 52 i=3,np-1
      av(i)=(-x(i-2)+2*x(i-1)-2*x(i+1)+x(i+2))/(del**3)
52    continue
      av(np)=0
      av(np+1)=0
      do 53 i=1,np+1
      d(i)=(i-1)*del
53    continue
      do 60 i=1,np+1
      write(*,61) d(i),am(i)
60    continue
61    format(2x,2f9.3)
      stop
      end

      subroutine gauss(a,p,x,n)
      double precision a(200,200),p(200,1),x(200)
      do 200 i=1,n
         do 200 j=1,n
            if (i.ne.j) then
            p(j,1)=p(j,1)-a(j,i)*p(i,1)/a(i,i)
            do 100 k=n,1,-1
100            a(j,k)=a(j,k)-a(i,k)*a(j,i)/a(i,i)
            endif
200         continue
      do 300 i=1,n
300   x(i)=  p(i,1)/a(i,i)
      return
      end
```